A NEW STRESS ANALYSIS CRITERIA FOR ROLLER CHAIN PLATE

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ABSTRACT

The design and applications of roller chains in engineering problems have been important subject for researchers as well as manufacturers for decades.
The suitability of a chain drive, for a given application, does not depend on the chain's breaking force, but solely on its fatigue strength and resistance to wear. With chain drives, a purely static load does not exist. When considering the load status of a chain link, load variations take place continuously in all individual phases of a cycle.
In this study, theoretically, an optimum design criteria has been developed in order to have uniformly distributed stress which has been less than the allowable stress of the material on chain plate.
The stresses have been analyzed due to distributed external load, instead of simple concentrating tension force. Some changes on the shape (classical geometry) have been found for material saving and increasing the strength.

1. INTRODUCTION

Belts and chains, as known flexible machine elements, are used mostly for power transmission in engineering applications. They usually take place of a group of shafts, gears and bearings, and give good approach to engineer to simplify the design. The flexibility of these machine elements can easily damp-out the shock and vibration effects.
When considering a chain link, a force variations take place in different phases of a cycle. Force takes a maximum value at position, then gradually reduce across several chains links when the chain arrives on the driving chain wheel and becomes zero at a position. Between these positions, there is a smooth rise to the value of initial tension and then again a gradual reduction to zero.
The maximum value as a concentrating force of this force variation has been taken into account in classical chain plate design criteria. In this approach there is a certain stress variation over the chain plate which has constant thickness.
In this study, the distributed force on the chain bearing is taken into account and design optimization has been made in order to have uniformly distributed stress over the chain plate.

Stress Analysis on Chain Plate

Classical design of chain plate does not give uniform stress distribution. Optimization of dimensions and geometry, for uniform stress distribution on the chain plate due to maximum force acting during the working conditions, becomes very important on chain design.

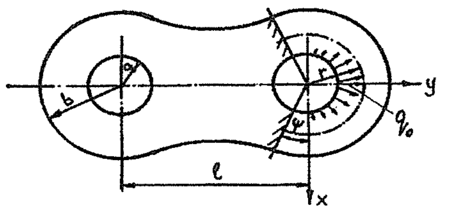

Figure 1. Geometry and loading of chain plate

In the classical design, external force is assumed to be as a simple tension force on the axisymmetrical direction. This assumption does not reflect the force application on chain link in practice. Because there is a distributed load along the half area of chain bearing hole [1, 2]. The form of this distributed force can be taken as (Figure 1):

$$q = q_0 . \sin\alpha \tag{1}$$

where q_0 can be found, in terms of tension force P, by equilibrium of forces on axial direction as in the following form , figure 2.

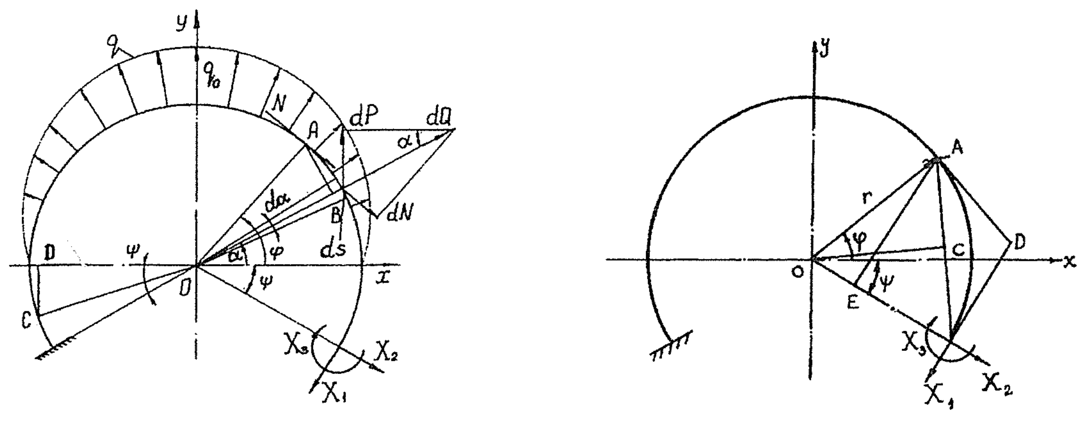

Figure 2. Pressure variation on bearing hole of chain plate and schematic geometry

$$P = 2\int_0^{\pi/2} dQ.\sin\alpha$$

and

$$q_0 = \frac{2}{\pi . r} P . \tag{2}$$

The central part of chain plate is more rigid comparing to circular part. Thus, the model of circular part can be taken as curved beam which is fixed from both ends, figure 1. This assumption makes the problem as statically undetermined of third degree.
In order to solve this third order statically undetermined problem, one of the fixed support removed and X_1, X_2, X_3 are taken as support reactions. Here, X_1 is axial force, X_2 is shear force and X_3 is bending moment at the support.
There are different methods to find these unknowns. If the Force Method is used, the following set of equations can be written as in [3]:

$$\begin{aligned} \delta_{11}X_1 + \delta_{12}X_2 + \delta_{13}X_3 &= -\delta_{1p}, \\ \delta_{21}X_1 + \delta_{22}X_2 + \delta_{23}X_3 &= -\delta_{2p}, \\ \delta_{31}X_1 + \delta_{32}X_2 + \delta_{33}X_3 &= -\delta_{3p}. \end{aligned} \tag{3}$$

Here;

δ_{ik} are known as flexibility coefficients which are displacements on direction of force X_i by application of unit force on direction of force X_k,

δ_{ip} are displacements on direction of force X_i by application of force P.

δ_{ik} can be written as in the following form;

$$\delta_{ik} = \int_s \frac{M_{xp} M_{xi}}{EI_x} ds + \int_s \frac{N_P N_i}{EA} ds. \tag{4}$$

Here, A is cross sectional area of beam, M_{XP} and N_P , are bending moment and axial force due to force P, M_{xi} and N_i are moment and axial force due to unit force on direction, E is Young Modulus of material and I_x is moment of inertia for cross section area. Second term in relation (4) can be neglected, because of order of displacements for axial force is much smaller comparing to order of displacements for bending moments.
In order to solve the set of equation in (3), for the given loading conditions, figure 2, bending moments for different sections are written as;

Part I : In this part, $(-\psi \le \varphi \le 0)$, axial force $N_P^I = 0$ and bending moment $M_P^I = 0$
Part II : In this part, $(0 \le \varphi \le \pi)$,

$$dM_P^{II} = -AB.dQ = -r^2 q_0 \sin\alpha . \sin(\varphi - \alpha) d\alpha. \tag{5}$$

For a cross section in part II, bending moment will be as;

$$M_P^{II} = \int_s dM_P^{II}$$

or

$$M^{II}{}_P = -\int_0^{\varphi} q_0 r^2 \sin\alpha \sin(\varphi-\alpha) d\alpha = -q_0 r^2 \int_0^{\varphi} \sin\alpha(\sin\varphi\cos\alpha - \cos\varphi\sin\alpha) d\alpha$$

and after necessary calculations,

$$M^{II}{}_P = P\frac{r}{\pi}(\varphi.\cos\varphi - \sin\varphi). \tag{6}$$

Normal force on a cross section, Figure 2, can be written as;

$$dN_P^{II} = dQ.\sin(\varphi-\alpha) = \frac{2P}{\pi}\sin\alpha.\sin(\varphi-\alpha).d\alpha = -\frac{P}{\pi}[\cos\varphi - \cos(2\alpha-\varphi)]d\alpha,$$

and by taking integral, axial force will be;

$$N_P^{II} = \int_0^{\varphi} dN = -\frac{P}{\pi}\int_0^{\varphi}[\cos\varphi - \cos(2\alpha-\varphi)]d\alpha = -\frac{P}{\pi}(\varphi.\cos\varphi - \sin\varphi),$$

$$N_P^{II} = -\frac{P}{\pi}(\varphi.\cos\varphi - \sin\varphi) \tag{7}$$

Part III : In this part, $(\pi \le \varphi \le \pi + \psi)$,

$$M^{III}{}_P = -P.OD = -P.r\cos(\varphi - \pi),$$

$$M^{III}{}_P = \Pr\cos\varphi, \tag{8}$$

Bending moment at section defined by angle φ, due to force X_1 will be in the following form (Figure 2);

$$M_1 = X_1.AD \tag{9}$$

By using necessary geometrical relations it can be written as;

$$M_1 = X_1.r.[1 - \cos(\varphi + \psi)] \tag{10}$$

for $X_1 = 1$, we find,

$$M_1 = r.[1 - \cos(\varphi + \psi)]\ . \tag{11}$$

The bending moment due to force X_2 is

$$M_2 = - X_2 \,.\, AE \,. \tag{12}$$

From Figure 2, we find AE as in the following form:

$$AE = r \sin\left(\varphi + \psi\right)$$

then we get,

$$M_2 = -X_2 . r \sin(\varphi + \psi), \tag{13}$$

for $X_2 = 1$,

$$M_2 = -r . \sin(\varphi + \psi) \,. \tag{14}$$

The bending moment at section defined by angle φ due to X_3 is,

$$M_3 = X_3 \tag{15}$$

And for $X_3 = 1$,

$$M_3 = 1 \tag{16}$$

In order to find coefficients δ_{ik} in equations (3), the first term of relation (4) will be used

and then,

$$\delta_{11} = \frac{1}{EI} \int_{-\psi}^{\pi+\psi} M_1^2 ds,$$

using relation (11) we get,

$$EI.\delta_{11} = r^3 \int_{-\psi}^{\pi+\psi} [1 - \cos(\varphi + \psi)]^2 d\varphi$$

and

$$\delta_{11} = \frac{r^3}{EI} \left\{ \frac{3}{2}(\pi + 2\psi) + 2\sin 2\psi + \frac{1}{4}\sin 4\psi \right\}. \tag{17}$$

Similarly,

$$\delta_{22} = \frac{r^3}{2\,EI} \left[\pi + 2\psi - \frac{1}{2}\sin\, 4\psi \right]. \tag{18}$$

and

$$\delta_{33} = \frac{r}{EI}(\pi + 2\psi) \tag{19}$$

It is known that $\delta_{12} = \delta_{21}; \delta_{23} = \delta_{32}$ and $\delta_{13} = \delta_{31}$, so the rest of the coefficients can be found as in the following form;

$$\delta_{13} = \frac{r^2}{EI}[\pi + 2\psi + \sin 2\psi], \tag{20}$$

$$\delta_{12} = -\frac{r^3}{4EI}(3 + 4\cos 2\psi + \cos 4\psi). \tag{21}$$

$$\delta_{23} = -\frac{2r^2}{EI}\cos\varphi. \tag{22}$$

In order to find coefficients δ_{1p}, δ_{2p} ve δ_{3p}, we use relations (6), (8) and (11), and find

$$EI\delta_{1p} = \int_0^{\pi} M_p^{II} M_1.ds + \int_{\pi}^{\pi+\psi} M_p^{III} M_1 ds$$

$$\delta_{1p} = -\frac{1}{EI}\left\{\frac{q_0.r^4}{8}\left(16 + 3\pi\sin\psi + \pi^2\cos\psi\right) + \frac{\Pr^3}{4}\left(3\sin\psi + \sin 3\psi + 2\psi\cos\psi\right)\right\}.$$

or

$$\delta_{1p} = -\frac{P.r^3}{4EI}\left[\frac{16}{\pi} + 6\sin\psi + \sin 3\psi + (\pi + 2\psi).\cos\psi\right]. \tag{23}$$

Similarly,

$$EI\delta_{2p} = \int_0^{\pi} M_p^{II} M_2.ds + \int_{\pi}^{\pi+\psi} M_p^{III} M_2.ds ,$$

and

$$EI\delta_{2P} = -\frac{\Pr^3}{4}[\pi\sin\psi - 3\cos\psi + 2(\psi + \sin 2\psi)\sin\psi] \tag{24}$$

$$EI\delta_{3P} = \int_0^{\pi} M_P^{II} M_3.ds + \int_{\pi}^{\pi+\psi} M_P^{III} M_3.ds,$$

or

$$EI\delta_{3P} = -\Pr^2\left(\frac{4}{\pi} + \sin\psi\right). \tag{25}$$

Since the coefficients of equations in (3) are known, then by using Cramer's method, the solutions are found as in the following way;

$$X_1 = \frac{D_1}{D}, \quad X_2 = \frac{D_2}{D}, \quad \text{and} \quad X_3 = \frac{D_3}{D}. \tag{26}$$

where, D, D_1, D_2 and D_3 are determinants of coefficients as,

$$D = \begin{vmatrix} \delta_{11} & \delta_{12} & \delta_{13} \\ \delta_{21} & \delta_{22} & \delta_{23} \\ \delta_{31} & \delta_{32} & \delta_{33} \end{vmatrix}, \qquad D_1 = \begin{vmatrix} -\delta_{1p} & \delta_{12} & \delta_{13} \\ -\delta_{2p} & \delta_{22} & \delta_{23} \\ -\delta_{3p} & \delta_{32} & \delta_{33} \end{vmatrix},$$

$$D_2 = \begin{vmatrix} \delta_{11} & -\delta_{1p} & \delta_{13} \\ \delta_{21} & -\delta_{2p} & \delta_{23} \\ \delta_{31} & -\delta_{3p} & \delta_{33} \end{vmatrix}, \qquad D_3 = \begin{vmatrix} \delta_{11} & \delta_{12} & -\delta_{1p} \\ \delta_{21} & \delta_{22} & -\delta_{2p} \\ \delta_{31} & \delta_{32} & -\delta_{3p} \end{vmatrix} \tag{27}$$

X_1, X_2 and X_3 are found by these relations and then substituting these values in (10), (12) and (15), normal force and bending moment in two parts of the structure are found as in the following form:

Part I: $(-\psi \le \varphi \le 0)$

$$\begin{aligned} N_1 &= X_1.\cos(\varphi+\psi) + X_2.\sin(\varphi+\psi), \\ M_1 &= X_3 + r.\{X_1[1-\cos(\varphi+\psi)] - X_2.\sin(\varphi+\psi)\}. \end{aligned} \tag{28}$$

Part II: $(0 \le \varphi \le \pi)$

$$N_2 = X_1.\cos(\varphi+\psi) + X_2.\sin(\varphi+\psi) - \frac{P}{\pi}(\varphi.\cos\varphi - \sin\varphi),$$

$$M_2 = X_3 + r.\{X_{1.}[1-\cos(\varphi+\psi)] - X_2.\sin(\varphi+\psi)\} + \frac{P}{\pi} r.(\varphi.\cos\varphi - \sin\varphi) \tag{29}$$

Since the system is symmetric, then it is not necessary to write the relations for the third part.
Normal stress at a point of a cross section can be written in terms of normal force and bending moment as;

$$\sigma = \frac{M}{A.e} \frac{y_1}{(r_0 + y_1)} + \frac{N}{A}. \tag{30}$$

Here, r_0 is the radius which defines the neutral plane and can be found by using the following relation (Figure 3);

$$r_0 = \frac{h}{\ln \dfrac{r + h/2}{r - h/2}} \tag{31}$$

and the distance, between geometric center G and point O on the neutral plane, e is

$$e = r - r_0 \tag{32}$$

and A is the cross sectional area (A = t . h) , h is the width of plate (h = b-a), t is thickness of the chain plate and y_1 is ordinate of point C (Figure 3).

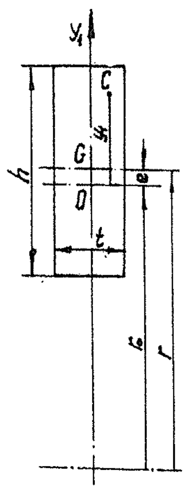

Figure 3. Geometry of cross section

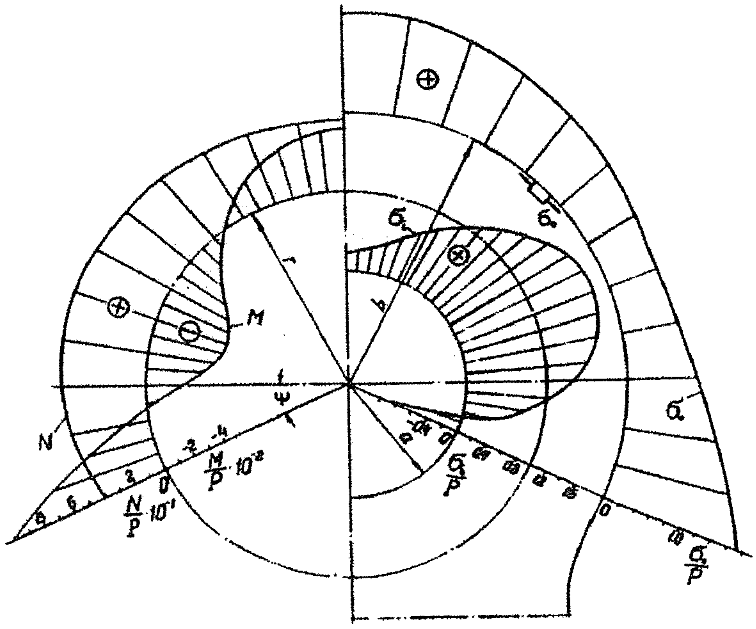

Figure 4. Normal force, bending moment and normal stress variations

Variation of bending moment, normal force and normal stress are shown in figure 4. The normal stresses have been calculated for the example on which numerical values are $\psi = 25$, r = 1.2cm., P=10 kN., and h = a – b = 1 cm. In order to have uniform stress distribution all over the chain plate;

$$\frac{M}{A.e}\frac{y_1}{(r_0 + y_1)} + \frac{N}{A} \leq \sigma_p,$$

or

$$\frac{M}{h.t.e}\frac{y_1}{(r_0 + y_1)} + \frac{N}{h.t} \leq \sigma_p \tag{36}$$

where σ_p is the stress for elastic limit.

If we keep h as constant find t for uniform stress, then

$$t = \frac{1}{h\sigma_p}\left[\frac{M}{e}\frac{y_1}{(r_0 + y_1)} + N\right] \tag{33}$$

This relation defines the variation of thickness t for which the stress over the circular part of chain plate be uniform and this variation shows a hyperbolic form.

2. RESULTS AND DISCUSSION

Variation of normal force, bending moment and normal stress along the curved beam model have been shown in figure 4. For the numerical example, (for $\psi = 25$) the maximum bending moment occurs at cross section, which is defined by $\varphi = 18$, The other critical cross sections are fixed support where, $\varphi = -25$ and the section for $\varphi = 90$. Figure 5 shows the form of thickness variation at these critical sections for uniform stress distribution which are drawn, by using eqn. 33, for $\sigma_p = 18KN/cm^2$.

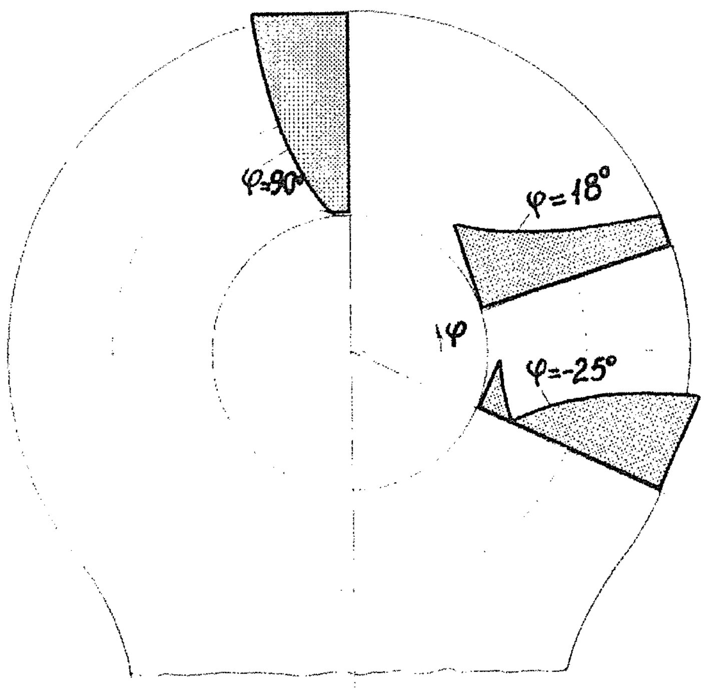

Figure 5. Variation of thickness for uniform stress distribution at critical cross sections for $\varphi = -25$.

Superposition of these thickness variation all over the plate is given in figure 6. Suggested form of thickness, as optimization form, is drawn on the top of the figure.
Optimization of geometry of chain plate will give uniform stress distribution on all over the curved part. Variable thickness design will be more reliable, especially for fatigue problem. As a next step of this research will be fatigue analysis and necessary design modifications of roller chain plate.

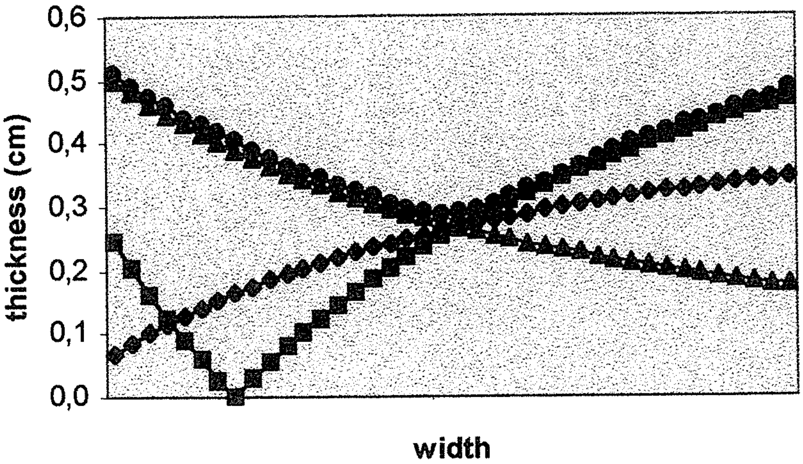

Figure 6. Superposition of thickness forms for uniform stress distribution
(■: $\varphi = -25^\circ$, ▲: $\varphi = 18^\circ$, ◆: $\varphi = 90^\circ$, and ●: Suggested thickness form)

3. ACKNOWLEDGMENT

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Ziya ABDULALİYEV received his B.Sc. degree from Azebaycan Technical University in 1962. Then he worked as research assistant at Science Academy, Moscow, USSR, and received his Ph.D. from Science Academy in 1976. He has been at University of Physics Engineering in Moscow, more than 14 years. Now he is visiting Professor at İstanbul Technical University, Mechanical Engineering Department. His major research areas are design and stress analysis. He is expert especially on photo elasticity.

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