Comprehensive Dynamic Zoning Algorithms

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ABSTRACT

During Elevcon'95 held in Hong Kong, a paper was presented, that brought the term "dynamic zoning" to the world. "Dynamic zoning" was an improved version of the "channelling" technique developed almost a decade ago. In Volume 1 of International Journal of Elevator Engineering, that paper on dynamic zoning was further elaborated and more details were included. However, up to this moment, the consideration has been with up-peak and down-peak conditions only. In this paper, the concept of dynamic zoning has been extended to include random interfloor traffic conditions as well while up-peak and down-peak become special cases. The most general traffic condition of interfloor traffic with unequal floor demand and population is employed to arrive at an appropriate "equivalent round trip time" for each car with an assigned zone. A cost function based on the optimal equivalent round trip time of all cars is used to select the best zoning using a genetic algorithm due to its high speed of convergence. Hence, the name "comprehensive" is adopted.

1 INTRODUCTION

It is generally aware that lift traffic patterns are changing significantly during the daily operations, from up-peak, down-peak to peak interfloor and even off-peak etc. Conventional lift system design has had much emphasis on up-peak traffic. Hence, the design of a good lift traffic controller becomes a very important job if the target of optimal control is required whole day long. Here, optimal control refers to highest handling capacity and shortest waiting time and travelling time of passengers. Zoning is a classical way to achieving part of the aims to improve the performance of an lift system.

In high rise buildings, lifts are normally clustered into zones to reduce number of stops and total journey time [1]. The floors served are usually adjacent, although some buildings may have split subzones where the occupants of each subzone are associated with each other and can be expected to generate some interfloor movements. A group of cars serving a high rise zone also provides service to and from the main terminal, travelling express between this floor and the lowest floor in the zone, known as the express zone terminal. A major advantage of zoning is the increase of the lift system handling capacity. At present, zoning in a high rise building may either be static or time-scheduled. Static zoning refers to the permanent assignment of a group of cars to service a number of floors usually adjacent in the building. Time-scheduled zoning refers to temporary zoning of the building during a pre-scheduled period of time during the day. The period usually coincides with the peak traffic situation or rush hours and the lifts service all floors outside this period.

The merit of zoning is not controversial. Actually, it has been implemented in lots of commercial and domestic buildings. However, the existing control patterns of zoning are either pre-determined or fixed on a time-schedules basis, or in other words, they are not adaptable to the real time traffic patterns. This shortfall initiates our research into the idea of dynamic zoning. The concept of primitive dynamic zoning for up-peak only has been developed [2] so that lifts make less stops per round trip and cars return to lobby faster. However, that system was not adaptable to the changing traffic flow patterns and the assignment of floors for channelling was not totally dynamic. The theory and implementation of "Dynamic Zoning" on up-peak and down-peak conditions were published before [3,4] and it was shown that substantial improvement on both the round trip time and handling capacity could be achieved.

2 A QUICK REVIEW ON DYNAMIC ZONING OF 1ST GENERATION

The 1st generation refers to a system handling up-peak and down-peak traffic conditions only. It is assumed that the traffic demand of each floor is known. A commercial building with N number of floors, excluding main terminal (MT = 0/F), is considered. Within the 5-minute duration of up-peak, total demand of the building is U and the demand of the kth floor is U_k, k=1,..,N. The building is served by a group of m numbers of lift cars and no overlapping in service is ensured. Naturally, m cars can divide the building into m number of zones. A clearer picture of the zoning arrangement is shown below:

1st car serving 0/F(MT), 1/F, ... , n_1/F;

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jth car serving 0/F(MT), $(n_{j-1}+1)$/F, ... , n_j/F (j = 2, ... , m-1);

•

mth car serving 0/F(MT), $(n_{m-1}+1)$/F, ... , N/F.

A strict rule is: $0 < n_1 < n_2 \bullet \bullet < n_j < \bullet \bullet < n_{m-1} < N$. The objective of dynamic zoning is to find an optimal solution of the (m-1) number of n's during the two modes of traffic flow. The round trip time of the jth car is denoted by RTT_j [1]:

$$RTT_j = 2\, H_j\, t_v + (\, S_j + 1\,)\, t_s + 2\, p_j\, t_p \tag{1}$$

The mathematical problem then becomes the minimisation of either of the following two cost functions:

$$\underset{n_1,\dots,n_{m-1}}{Min}\ \sigma\left(RTT_j \big|_{j=1}^{m} \right) = \sqrt{\frac{1}{m}\sum_{j=1}^{m}\left(RTT_j - \overline{RTT}\right)^2}$$

$$\text{where } p_j = \begin{cases} \dfrac{U_j\, RTT_j}{300} & \text{if } \dfrac{U_j RTT_j}{300} < CC_j \\ CC_j & \text{if } \dfrac{U_j RTT_j}{300} \geq CC_j \end{cases} \tag{2}$$

$$U_j \triangleq \sum_{k=n_{j-1}+1}^{n_j} U_k \quad ; \quad \overline{RTT} = \frac{1}{m}\sum_{j=1}^{m} RTT_j$$

$$\min_{n_1, \ldots, n_{m-1}} \sum_{j=1}^{m} (U_j - HC_j)^2 \quad \text{where} \quad HC_j \triangleq \frac{300}{RTT_j} (0.8CC_j) \tag{3}$$

CC_j is the contract capacity of the jth car. Here, p_j is the number of passengers in each round trip and it is not simply equal to 80% of the contract capacity because it very much depends on the number of floors being serviced by the jth car. If the arrival rate exceeds the handling capacity, the number of passengers inside each car will be restricted to the overall contract capacity of the car.

Under up-peak traffic conditions, H_j(up-peak) and S_j(up-peak) can be formulated for each zone in accordance with CIBSE guide [5] for unequal floor population using rectangular probability distribution function, i.e. the arrival rate of passengers at the main terminal is uniform, and regular respect to time.

$$H_j \ (up\text{-}peak) = n_j - \sum_{k=n_{j-1}+1}^{n_j - 1} \left(\sum_{l=n_{j-1}+1}^{k} \frac{U_l}{U_j} \right)^{p_j} \qquad n_0 = 0 \tag{4}$$

$$S_j \ (up\text{-}peak) = [n_j - n_{j-1}] - \sum_{k=n_{j-1}+1}^{n_j} \left(1 - \frac{U_k}{U_j} \right)^{p_j} \tag{5}$$

Under down-peak traffic conditions, Strakosch's approach is adopted [1]. p_j (down-peak) is assigned to be the contract capacity of the lift car because the duration of down-peak tends to be shorter and the demand is much higher. Then, most lift cars will be fully loaded and each lift tends to reach the top floor of its servicing zone for each round trip until passengers of the higher floors have all been serviced.

$$\begin{aligned} p_j \ (down\text{-}peak) &= CC \quad ; \quad H_j \ (down\text{-}peak) = n_j \\ S_j \ (down\text{-}peak) &= \frac{3}{4} S_j \ (up\text{-}peak) \end{aligned} \tag{6}$$

3 THE COMPREHENSIVE ALGORITHMS OF DYNAMIC ZONING

3.1 Equations of the General Traffic Conditions

The method adopted follows Alexandris' development as detailed in Section 7.1.3 of Barney's book [1]. They are listed here for the sake of completness. A building with N floors above the main terminal is considered. The total population of the building is given by U while the population of the ith and jth floors is given by U_i and U_j respectively. The rate of passenger arrivals to the ith floor is given by λ_i and the lift cycle time is given by T. Here, T is constantly set to 30 seconds which has been the conventional guideline for waiting time in a middle class commercial building. The probability that no call from the ith floor to the jth floor is given by:

$$pr_{ij} = e^{-\lambda_i T \frac{U_j}{U}} \quad for\ i, j = 0, 1, 2, \cdots, N \tag{7}$$

Furthermore, $pr_{ii} = 1$ and that implies it is very certain that no call will be registered from the ith floor to itself. The assumption that all passengers going to the main terminal will leave the building has to be modified in our algorithm. The up-peak and down-peak conditions were handled in our last paper [4] and summarised in the last section. The major achievement of this paper is to take care of the random interfloor traffic. Then, all passengers from the ith floor going to the main terminal desire to travel to the jth floor (destination) which is not belonging to the same zone as the departing floor, the ith floor. In other words, the reason why passengers travel to the main terminal is to change the lift to go to a floor of another zone. Furthermore, it is assumed that lifts will only travel round trips, i.e. from main terminal to the highest reversal floor and then back to the main terminal with a total number of S stops. Let A_k^j be the event that passengers enter the lift at the kth floor, k = 0, 1, ..., N while the jth floor is the highest reversal floor. Assuming that passengers enter the car at the ith floor, the probability, π_{ij}, that the jth floor will be the highest reversal floor is given by:

$$\pi_{ij} = \left\{ \begin{array}{lc} (1 - pr_{ij}) \prod_{k=j+1}^{N} pr_{ik} & i < j \\ \prod_{k=j+1}^{N} pr_{ik} & i = j \\ 1 & i = j = N \\ 0 & i > j \end{array} \right\} = pr(A_i^j) \tag{8}$$

$$i, j = 0, 1, ..., N$$

As the events, A_0^j, ..., A_N^j are not mutually exclusive but independent of each other, Poincare's result can be applied and the probability, π_j, that the jth floor will be the highest floor can be given by:

$$\begin{aligned} \pi_j = pr\left(\bigcup_{k=0}^{N} A_k^j \right) &= \sum_{k=0}^{N} pr\left(A_k^j \right) - \sum_{\substack{k_1, k_2 = 0 \\ k_1 < k_2}}^{N} pr\left(A_{k_1}^j \right) pr\left(A_{k_2}^j \right) \\ &+ \sum_{\substack{k_1, k_2, k_3 = 0 \\ k_1 < k_2 < k_3}}^{N} pr\left(A_{k_1}^j \right) pr\left(A_{k_2}^j \right) pr\left(A_{k_3}^j \right) \\ &+ \dots + (-1)^N \prod_{k=0}^{N} pr\left(A_k^j \right) \end{aligned} \tag{9}$$

To be simple, the individual probability that the jth floor is the highest floor when passengers enter the lift at the k_i th floor is given by:

$$\pi_{k_i j} = pr\left(A^{j}_{k_i}\right) \tag{10}$$

The highest reversal floor, H, is then given by:

$$\begin{aligned} H &= \sum_{j=1}^{N} j\,\pi_j \\ &= \sum_{j=1}^{N} j \left(\sum_{k=0}^{N} \pi_{kj} - \sum_{\substack{k_1, k_2 = 0 \\ k_1 < k_2}}^{N} \pi_{k_1 j}\,\pi_{k_2 j} + \ldots + (-1)^N \prod_{k=0}^{N} \pi_{kj} \right) \end{aligned} \tag{11}$$

Next, the expected number of stops within a round trip is estimated. The probability, W_{ki}, that at least one passenger wants to go to the ith floor from the kth floor is given by:

$$W_{ki} = 1 - pr_{ki} = 1 - e^{-\lambda_k T \frac{U_i}{U}} \tag{12}$$

The probability, S_i, that the ith floor is a stopping floor is given by:

$$S_i = \sum_{k=0}^{N} W_{ki} - \sum_{\substack{k_1, k_2 = 0 \\ k_1 < k_2}}^{N} W_{k_1 i}\,W_{k_2 i} + \ldots + (-1)^N \prod_{k=0}^{N} W_{ki} \tag{13}$$

The expected number of stops, S, within a round trip is then given by the summation of all S_i's, i.e.

$$\begin{aligned} S &= \sum_{i=0}^{N} S_i \\ &= \sum_{i=0}^{N} \left(\sum_{k=0}^{N} W_{ki} - \sum_{\substack{k_1, k_2 = 0 \\ k_1 < k_2}}^{N} W_{k_1 i}\,W_{k_2 i} + \ldots + (-1)^N \prod_{k=0}^{N} W_{ki} \right) \end{aligned} \tag{14}$$

It should be noted that the S calculated in equation (14) only takes care of the stops which are destination floors and the maximum value of S is N. A floor where nobody alights the car but lots of passengers board the car has not been included. Therefore, the ith floor whose λ_i is not equal to zero will contribute an additional value of 1 to S so that the maximum value of S can be up to 2 N when the lift car stops in both the up-trip and down-trip. After H and S have been estimated, the "equivalent round trip time" of a car can be calculated.

3.2 The Equivalent Round Trip Time

A building, of N storeys high excluding the main terminal, with a total population, U, is served by m number of lifts. The population of the ith floor is given by U_i, as used before. The demand of the ith floor is give by λ_i and the lift cycle time is given by T.

In other works, the number of passengers asking for service at the ith floor is given by $\lambda_i T$ within a period of T seconds. If this ith floor is served by one lift only, the total demand of this floor on that lift will be $\lambda_i T$. If this ith floor is served by n number of lifts, the demand on each lift will be $1/n\ \lambda_i T$. Therefore, overlapping zones assigned to two or more lifts are allowed, thus giving even more flexibility. The distribution of U_i has been known during the design stage as the owner needs to identify the utilisation of the building. λ_i can be obtained in different ways. A statistical analysis can be carried out by sensing the change of weight of a lift car stopping at the ith floor. Normally, it is assumed that each passenger has an average weight of around 75 kg and alighting passengers will go first followed by boarding passengers every time when a car stops. Another more effective way is to make use of computer vision [6] which is able to estimate the number of passengers waiting at the lobby of each landing under a real-time mode.

Two more parameters that need to be identified are the population and demand at the main terminal. Theoretically, U_0 is undefined because it is impossible to assume a population for the public. However, pr_{i0} for i=1, ..., N can be defined. General traffic patterns can be categorised into three modes, namely up-peak, down-peak and interfloor, by using an artificial neural network [7]. For up-peak mode of traffic, all pr_{ij} will be set to 1 if $i \neq 0$. For down-peak mode of traffic, all pr_{ij} will be set to 1 if $j \neq 0$. For interfloor mode of traffic, it is assumed that nobody leaves or enter the building, i.e. all pr_{i0} will be set to 1. Hence, the passengers waiting for service at the main terminal will only be equal to the sum of all passengers from the upper floors whose destinations are not within the zone of their departing floors. Suppose the kth lift among the group of m lifts is assigned a zone of 0/F k_1/F, k_2/F until k_h/F, passengers to jth floor where $j \notin \{k_1, k_2, ..., k_h\}$ will contribute to λ_0. Then, λ_i^k T, demand of kth lift at the ith floor, can be divided into two groups. The first group includes passengers whose destination floors are within the zone and the second group includes passengers whose destination floors are outside the zone. For passengers in the second group, they need to take the lift down to the main terminal and have a "transit" to another lift serving the zoning including the destination floors. The probability, $(1 - pr_{ij}^k)$, that at least one passenger taking the kth lift travels from the ith floor to the jth floor is a good indicator to separate the passengers into two groups. PA_{i1}^k, number of passengers of group 1 departing the ith floor and PA_{i2}^k, number of passengers of group 2 departing the ith floor can be estimated below:

$$\underset{i \in \{k_1, ..., k_h\}}{PA_{i1}^k} = \frac{\sum_{j \in \{k_1, ..., k_h\}} (1 - pr_{ij}^k)}{\sum_{j=1}^{N} (1 - pr_{ij}^k)} \left[\lambda_i^k T \right] \tag{15}$$

$$\underset{i \in \{k_1, ..., k_h\}}{PA_{i2}^k} = \lambda_i^k T - PA_{i1}^k$$

Hence, $\lambda_0 T$ can be defined as the sum of all group two passengers of all floors running from 1/F to the N/F of all the m number of lifts.

$$\lambda_0 T - \sum_{k=1}^{m} \sum_{i \in \{k_1, \ldots, k_h\}} PA_{i2}^{k} \tag{16}$$

Without a real-time simulation, it is quite impossible to estimate the total number of round trips even if all U's and λ's are known. Here, what is required is just a suitable zoning arrangement. The contract capacity of each lift car is unlimited and it is assumed one round trip is enough to handle all waiting passengers, including those taking "transit" at the main terminal. In this way, the so-called "Equivalent Round Trip Time" of each car is calculated. The total number of passengers, PA^k, being handled by the kth car during an equivalent round trip, is given by the sum of all passengers leaving the floors inside the zone of the kth lift and the passengers from other zones into the kth lift's zone with "transit" at the main terminal.

$$PA^k = \sum_{i \in \{k_1, \ldots, k_h\}} \lambda_i^k T + \sum_{\substack{l=1 \\ l \neq k}}^{m} \sum_{q \in \{l_1, \ldots, l_h\}} \left[\lambda_q^l T \right] \frac{\sum_{i \in \{k_1, \ldots, k_h\}} \left(1 - pr_{qi}^{l} \right)}{\sum_{j=1}^{N} \left(1 - pr_{qj}^{l} \right)} \tag{17}$$

The $ERTT^k$, "equivalent round trip time" of the kth lift is given by:

$$ERTT^k = 2 H^k t_v + (S^k + 1) t_s + 2 PA^k t_p \tag{18}$$

All other figures follow conventional definitions.

4 OPTIMISATION BY GENETIC ALGORITHMS

It can be seen that the expressions of $ERRT^k$ consist of quite a number of probability based functions involving exponential functions and series, which are very complicated, making the whole function highly non-linear in characteristics. The conventional method of optimisation, such as Newton's Law and Slope of Steepest Descent etc., is considered not so appropriate and hence, a probability based optimisation method, i.e. Genetic Algorithm (GA), is employed. The cost function to be minimised for the m number of lifts is given by E such that:

$$E = \frac{1}{m} \sum_{k=1}^{m} \left(ERRT^k - \overline{ERRT} \right)^2$$
$$\overline{ERRT} = \frac{\sum_{k=1}^{m} ERRT^k}{m} \tag{19}$$

As GA is a relatively new method, the following session gives an introduction to it and then its application to our cost function is described.

4.1 Introduction to Genetic Algorithm

GA [8,9] is a method to help searching for the optimal solution to a complex problem, based on the principles of natural selection. It is basically an automated, intelligent approach to trial and error. Given specific formulae, rules, or arrangements to be optimized, a GA can arrive at a solution. The GA is also a model of machine learning which derives its behaviour from a metaphor of some of the mechanisms of "Evolution" in nature. The algorithm approaches the problem by using the principles of natural selection. This is done by the creation within a machine of "Population of Individuals" represented by "Chromosomes", in essence a set of character strings that are analogous to the base-4 chromosomes of the DNA. GAs allow us to solve problems that were previously considered too large or complicated and are useful in the very tricky area of nonlinear problems.

In most cases, long strings of '0' or '1' are used. First, a number of solutions (a population) are created by setting the parameters randomly throughout the search space. From this population of solutions, the worst members are discarded and the best members are then "crossed over" with each other by mixing the parameters (genes) from the most successful organisms, thus creating a new population. Additionally, every so often a gene will be altered slightly to produce a "mutation". As in real life, this type of continuous adaptation creates a very robust organism. The whole process continues through many "generations", with the best genes being handed down to future generations.

GAs operate on a coding of the parameters, rather than the parameters themselves. Just as the strands of DNA encode all the characteristics of a human in chains of amino acids, the parameters of the problem must be encoded in finite length strings. In practice, the genetic model of computation can be implemented by having arrays of bits or characters to represent the chromosomes. Simple bit manipulations allow the implementation of "Crossover", "Mutation" and other operations. Although a substantial amount of research has been performed on variable-length strings and other structures, the majority of work with GA algorithms is focused on fixed-length character strings. When the GA is implemented, the following cycle is adopted.

1. Initialize a usually random population of individuals;
2. Evaluate fitness of all initial individuals of population;
3. Test for termination criteria (time, fitness, etc.);
4. Select a sub-population for offspring production;
5. Recombine the "genes" of selected parents, i.e. simulation of crossover;
6. Perturbate the mated population stochastically, i.e. simulation of mutation;
7. Evaluate fitness of the resulted population;
8. Select the survivors from actual fitness;
9. Repeat 3 to 9 until termination criteria are satisfied.

4.2 Optimisation of Equivalent Round Trip Time

In our problem of dynamic zoning, the cost function E to be minimised is the sum of the variance of the equivalent round trip time among the m number of lifts. Each lift is assigned one zone and floors in one zone can be overlapping with floors of another zone. For simplicity, it is assumed that N = 10 and m = 3. Each zone is represented by a string, x_i i = 1, .., m of N binary variables, "1" implying that particular floor belongs to the zone and "0" vice versa. The following outlines the procedures of GA application:

1. The length of the chromosome string is N x m = 30;
2. The initial population of ten strings each of 30 binary variables is randomly generated;
3. The value of fitness, i.e. E, is calculated for each string in the initial population, e.g. '0100000000 1011100001 0000110011' decodes to 2/F belongs to the zone of lift no. 1; 1/F, 3/F, 4/F, 5/F and 10/F belong to the zone of lift no. 2; 5/F, 6/F, 9/F and 10/F belong to the zone of lift no. 3. Each of the ten strings produces a cost function.
4. The next generation of GA begins with reproduction. This is a simple copying operation, subject the values of fitness. Candidates with higher fitness, i.e. lower E, will have a greater probability of being reproduced in the succeeding generation and those with lower fitness will tend to die out. To visualise how this is done, the 'wheel of fortune' is a very good analogy where each string is assigned a segment size on the wheel. The size is proportional to its value of fitness. For each spin of the wheel, i.e. a trial, a string is copied to the breeding population if the wheel stops with the pointer in the segment of that string.
5. The crossover procedure operates on two most appropriately selected strings while the point of crossover is randomly selected, say 15, e.g.
   0100000000 1011100001 0000110011 mates with
   0100111000 1010101001 0011100001 produces

   0100000000 1011101001 0011100001 and
   0100111000 1010100001 0000110011
6. The last operation, i.e. mutation, is performed on a bit-by-bit basis. It is assumed that the probability of mutation is arbitrarily chosen as 0.01. With a total of 10 strings in the population, it should be expected that one mutation will occur in only one string for a period of ten generations (10x0.01 = 0.1). E.g. the mutation of the string "0100000000 1011100001 0000110011" becomes "0100001000 1011100001 0000110011".
7. The new population is now ready to be assigned values of fitness, and the crossover and mutation cycles are repeated.

5 CONCLUSION

The requirement for dynamic zoning has been discussed and a brief review of the 1st generation of dynamic zoning that could handle both up-peak and down-peak traffic conditions was reviewed. By making use a improved version of probability based functions, the highest reversal floor and the expected number of stops of an equivalent round trip of a lift car were estimated when the lift was assigned a dynamic zone and the traffic was the random interfloor type. Passengers travelling between zones needed to take a "transit" at the main terminal. Subject to the fact that the resultant model was highly non-linear, a probabilistic approach of optimisation, Genetic Algorithm, was employed to optimise on cost functions involving the variances of the equivalent round trip time. It is anticipated that this is a model generally enough to handle all different kinds of traffic. When more information about the passengers' intention and routes are know, this dynamic zoning model can be improved accordingly.

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7 REFERENCES

[1] G.C. Barney and S.M. Dos Santos, Ed. *Elevator Traffic Analysis, Design and Control*, IEE, Peter Peregrinus, London, 1985.

[2] B.A. Powell, "Important issues in up peak traffic handling", in G.C. Barney, Ed. *Elevator Technology 4*, IAEE, 1992, pp. 207-218.

[3] W.L. Chan and A.T.P. So, "Dynamic Zoning in Elevator Traffic Control", in G.C. Barney, Ed. *Elevator Technology 6*, IAEE, 1995, pp. 132-140.

[4] Chan W.L. and So A.T.P, "Dynamic zoning for intelligent supervisory control", *International Journal of Elevator Engineering*, Vol. 1, 1996, pp. 47-59.

[5] CIBSE, *CIBSE Guide D - Transportation Systems in Buildings*, CIBSE, 1993.

[6] So A.T.P., Chan W.L., Kuok H.S. and Liu S.K. "A comprehensive solution for computer vision based supervisory control", in Barney G.C. Eds. *Elevator Technology 5*, IAEE, 1993, pp. 203-211, reprinted in *Elevatori*, Vol. 23, No. 4, July/August, 1994, pp. 61-77.

[7] So A.T.P., Beebe J.R., Chan W.L. and Liu S.K. "An artificial neural-network based traffic patterns recognition system", *International Journal of Elevator Engineering*, Vol. 1, 1996, pp. 35-46.

[8] Schwefel H.P. eds. *Evolution and Optimum Seeking*, John Wiley & Sons, N.Y., 1995.

[9] Chong E.K.P. and Zak S.H. ed. *An Introduction to Optimisation*, John Wiley & Sons, N.Y., 1996, pp. 212-224.