TIME, DISTANCE, SPEED, ACCELERATION AND JERK IN ELEVATOR STARTING AND STOPPING

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ABSTRACT
The paper studies in detail the relationship of, for one part, the nominal speed and maximum allowable acceleration and jerk versus, for the other part, the times and distances needed for starting and stopping the elevator cars. In the first part the physical and mathematical aspects of the problem are exposed, showing the substantial intervening facts and trying to convey a deep and intuitive grasp on the subject. The second part develops practical methods to calculate the starting and stopping times and distances by means of tables and graphic curves. Finally the relationship between the above calculations and real world conditions is discussed, including considerations for comfort.

1. INTRODUCTION
During its trip, an elevator takes a succesion of instantaneous speeds which may be represented by a line in a cartesian plane, taking axis x to represent time and axis y for speed (Figure 1).

This curve defines completely all the kinematic variables of the trip: in each point its derivative **dy/dx = dv/dt** is precisely the instantaneous acceleration of the elevator and its second derivative $\mathbf{d^2y/dx^2}$ is the derivative of the acceleration with respect to time, usually called "jerk".

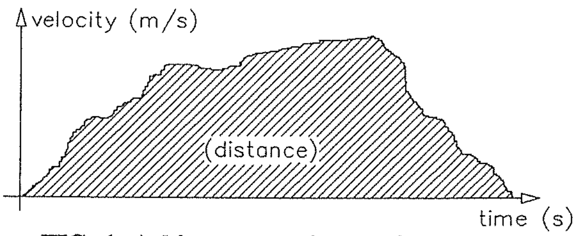


FIG. 1: Arbitrary velocity vs. time curve.

The integral $\int y\,dx$ between two points in time is the distance travelled by the car during that time.

As is well known, to achieve a satisfactory ride quality it is necessary that the variation of the car speed be progressive and as smooth as possible. But this smoothing may only be done at the cost of lengthening the time of travel, that is to say, by losing performance.

In the existing literature on this matter the expression "ideal kinematics" has been used to signify the most time-efficient trip curve, provided a minimum of comfort is achieved. Obviously this is just one way of being ideal. Another interesting ideal could be the maximum comfort trip, given a certain travel time limit. A balance must be struck between the two characteristics: duration of travel and comfort of ride (performance versus quality).

Unfortunately, due to the fact that the effects of acceleration and jerk values on comfort have not yet been well quantified, to get at this balance the designer has to resort to trial and error methods.

The best way is to first do the "homework", that is, calculate the effects of different choices of acceleration and jerk on the times and distances, and then, taking these effects into consideration, try them in the field and experience the comfort level achieved with them and decide their effectiveness.

2. "HOMEWORK"

We must define a set of speed-time curves that meet the desired travel performance with different values of acceleration and jerk.

The performance.- Simply stated, the optimization of performance means that the speed-time curve should be such that, covering a predefined area (which is the distance between the beginning and the end of the trip), its time length is minimised.

If the horizontal length of the figure must be a minimum while conserving its area, then its height should be a maximum. So the trip should be done at the maximum allowable speed. This speed is the rated speed of the lift and must be chosen while defining its main characteristics.

With no other constraints the minimum time would be given by a rectangular shape: that is the graphical equivalent of saying that for minimum travel time it should be done at rated speed all the time (Figure 2).

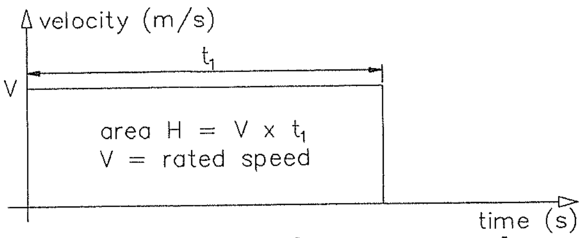


FIG. 2: v(t) curve for constant speed.

But this is physically impossible since it would need infinite acceleration to start and stop. Anyway the ride quality imposes a limit in the value of this acceleration. That means the curve may not have transitions that are steeper than the value corresponding to that acceleration limit (Figure 3).

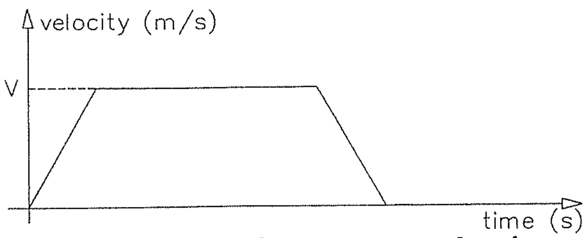


FIG. 3: v(t) curve for constant acceleration

Besides that, we know that even the change of acceleration can produce uncomfortable jerks, so the corners of the curve should be suitably rounded.

This is qualitatively the shape that the speed-time curve should have. This could be called the "typical curve" (Figure 4), since it is by far the most frequently followed in low and medium speed lifts, and we are going to recall briefly its main properties.

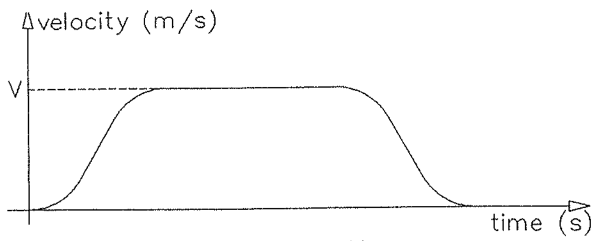


FIG. 4: Typical v(t) curve.

3. TYPICAL CURVE

The definition of the curve shown in Figure 5 has been made on the assumption that simply the maximum instantaneous values of acceleration and jerk define completely the comfort (ride quality), independently of sign, duration, repetition or any compound effect of both parameters together. So we try to apply in each moment the maximum allowable speed, acceleration and jerk, compatible with each other and with the foreseeable evolution of their values.

Even if there may be considered many possible variations of this curve, for the sake of clarity of ideas it is useful to analyze first this case.

Let us call **V**, **A** and **J** the maximum chosen values of speed, acceleration and jerk. Now we calculate this typical curve by succesive integrations:

- first we set the value **J** for maximum jerk: its integration is a ramp that represents the shape of the acceleration. The integration of this ramp is a parabola which re-

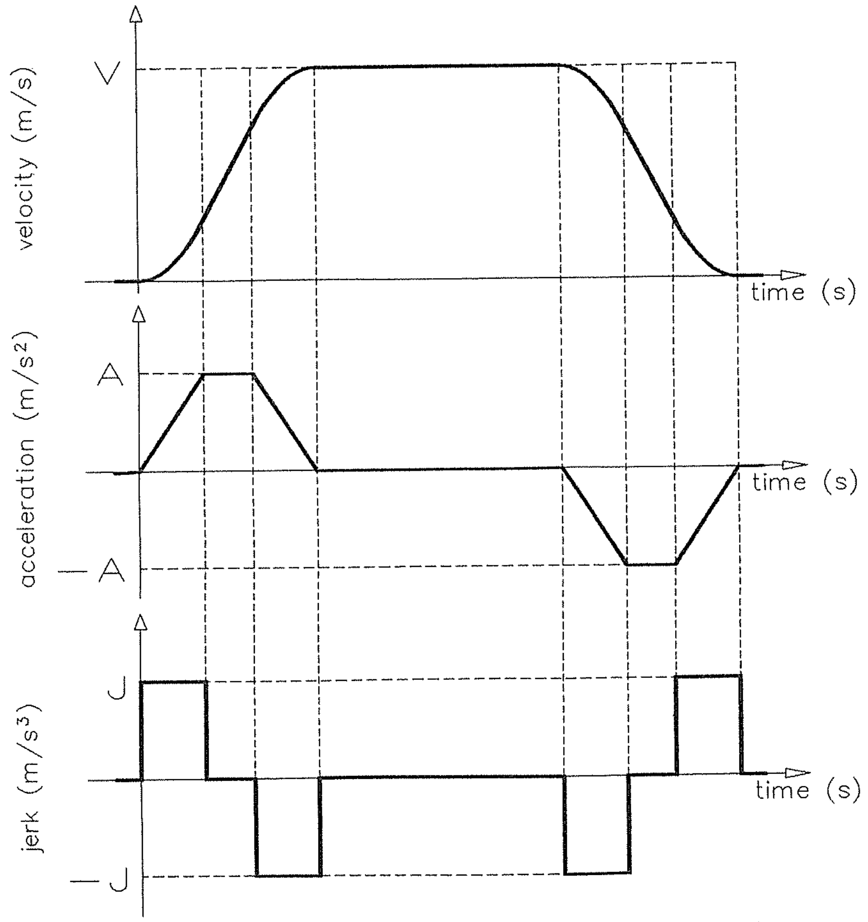


FIG. 5: Correspondence between jerk, acceleration and speed in a typical curve.

presents the shape of the speed. We follow this process up to the moment in which the maximum acceleration is achieved.

- then the acceleration is held constant by bringing the jerk to zero. Its integral (i.e. the speed) increases linearly up to the moment in which the acceleration must decrease to reach maximum speed at zero acceleration: this ends the constant acceleration part of the start.
- to decrease acceleration we produce a negative jerk -J whose integration is a negative ramp and so the instantaneous speed follows another parabolic segment up to its maximum value V.
- the following part is the constant speed travel.
- the deceleration part is simply symmetrical of the described starting part.

As a consequence of the way this curve is generated (see Figure 6), these symmetries are produced:

- as has been said, the whole speed-time curve is symmetrical with respect to the vertical in the middle of the trip.
- the increasing part of this curve (as is the decreasing part) is symmetrical with respect to its central point.
- the duration of each of its rounded parts is divided into two equal parts by the vertical passing through the vertex of the angle formed by the two straight segments at their tips (due to its parabolic shape).

As a consequence of these symmetries the total time necessary to make a trip of length H is the sum of three terms:

- the time that would be needed if along all of the trip the speed were constant and

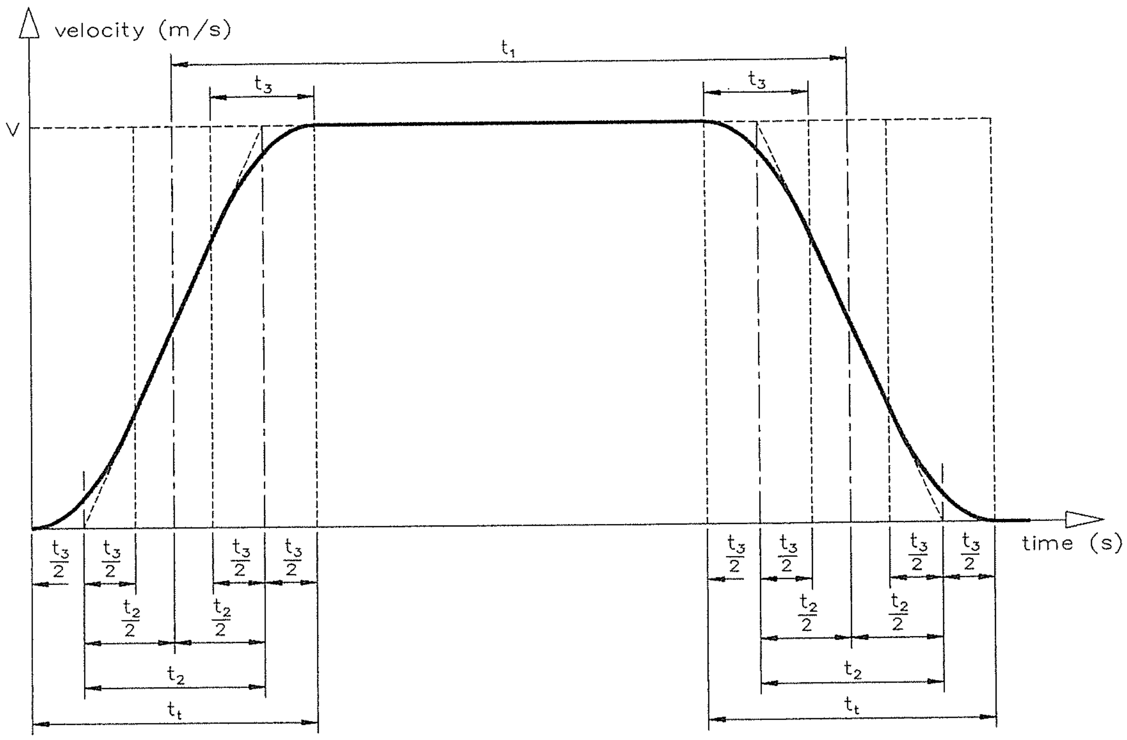


FIG. 6: Time intervals in a typical speed-time curve.

equal to the maximum speed V, i.e. $t_1 = H/V$ [1]

- the time that would be needed to achieve maximum speed if the acceleration value were constant and equal to its maximum A while accelerating (and, with a negative sign, while decelerating), that is to say, $t_2 = V/A$ [2]
- the time needed to achieve maximum acceleration (or deceleration) at maximum jerk J, that is $t_3 = A/J$ [3]

Thus, $t = t_1 + t_2 + t_3 = \frac{H}{V} + \frac{V}{A} + \frac{A}{J}$ (provided $t_1 \geq t_2 + t_3$ and $t_2 \geq t_3$) [4]

4. APPLICATION

This formula can be further simplified to give us a handy way to evaluate the performance of a prospective installation.

We may assume that the maximum acceleration will be near to 1 m/s^2 and the maximum jerk between 1.5 and 2.5 m/s^3.

Then, substituting in [4] we get **$t = H/V + V + 0.4..0.7 \approx H/V + V + 0.5$** [5] (provided we are using the SI unit system).

For elevator engineering purposes this formula is just an approximation, suitable for the first steps of the design. In the subsequent steps further detail should be considered.

As for ride quality, formula [5] shows that, in most situations (travels ranging over several floors), small variations in acceleration and/or jerk have little influence in the total time spent.

So, the practical question arises during the field adjustments:

- with the constraints of the furnished elevator, how can the best maximum acceleration and jerk values be chosen in order to optimize comfort?

This question has importance at starting and stopping, for at constant speed there is neither (intended) acceleration nor jerk.

5. STARTING AND STOPPING

Let us look again at the shape of the typical speed-time curve.

Due to the same symmetries stated before, the total transition time during starting or stopping is the sum of three parts:

$$t_t = \frac{t_3}{2} + t_2 + \frac{t_3}{2} = t_2 + t_3 = \frac{V}{A} + \frac{A}{J} \qquad [6]$$

For the transition at starting, one may choose a pair of values for J and A and calculate the corresponding time. In this case the choices are limited and it is relatively easy to make the trial and error task. Usually it is interesting to leave the transition time a little longer than strictly necessary since small increases in ride time result in considerable increases in comfort.

In the case of the stopping transition there appears one more constraint since the decelerating distance may also be a limiting factor. This is due to the fact that the position sensing elements employed for the control work better in certain zones. As a consequence it is often convenient that the decelerating transition be made in less than a specified distance. And as, obviously, shorter distance means less comfort, the problem may be stated as how to find out the optimal combination of acceleration and jerk for a fixed predetermined stopping distance.

It is easy to see, just by looking at the areas below the speed-time curve in that part, that the distance travelled during the starting transition is:

$$H_t = \frac{V}{2} t_t \qquad [7]$$

And as a consequence:

$$H_t = \frac{V}{2}\left(\frac{V}{A} + \frac{A}{J}\right) \qquad [8]$$

In the case we are considering: $H_t = k$ (constant)

Then $\frac{V}{A} + \frac{A}{J} = 2\frac{k}{V}$ (which is also a constant since V is the rated speed) [9]

Thus, we must devise a method to choose the most convenient combination with the fewest possible trials.

6. TRIAL METHOD

Taking as a base the rated speed V and the stopping distance H_t we can draw a curve in which J is in the x axis and A in the y axis, for the values that meet formula [9]:

$$\frac{V}{y} + \frac{y}{x} = 2\frac{H_t}{V} \qquad [10]$$

$$\frac{y}{x} = 2\frac{H_t}{V} - \frac{V}{y} \Rightarrow x = \frac{y}{2\frac{H_t}{V} - \frac{V}{y}} \qquad [11]$$

for example:

$$\left.\begin{aligned} V &= 1.6\frac{m}{s} \\ H_t &= 2m \end{aligned}\right\} \Rightarrow x = \frac{y}{2.5 - \frac{1.6}{y}}$$

This curve (FIG. 7a) shows at a glance all the different combinations of A and J that meet the desired stopping distance. It gives also a visual guidance on the trials that could be convenient to perform.

Thus, for each of the usual rated speeds we can draw a set of **A**, **J** curves, taking the stopping distance as a parameter (one for each of several possible curves) to choose in each case the most convenient values of said parameters (FIG. 7b).

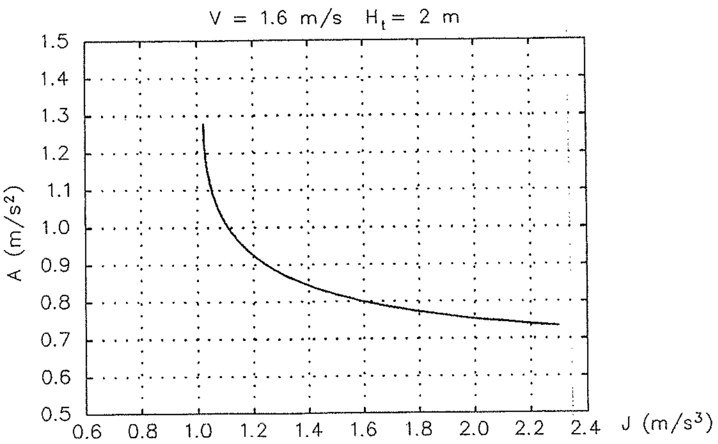


FIG. 7a: (A, J) curve for V = 1.6 m/s and H_t = 2 m.

7. MORE ACCURATE CALCULATION

In real elevators the transition from running speed to stop is accomplished in three steps:

In the first step speed is decreased from rated to levelling speed, (which is about 0.1 m/s) following the comfort criteria that have been considered above; in the second step the car runs for a short time at constant speed and in the third the braking is effected in some way or other.

The time spent and the distance travelled in the first step may be calculated the same way as before (see FIG. 8):

The time will be:

$$t_t = t_2 + t_3 = \frac{V - V_l}{A} + \frac{A}{J} \qquad [12]$$

The distance will be:

$$H_t = \frac{V + V_l}{2} t_t \qquad [13]$$

Then the **A** vs. **J** curve must meet:

$$\frac{V - V_l}{y} + \frac{y}{x} = 2\frac{H_t}{V + V_l} \qquad [14]$$

$$\frac{y}{x} = 2\frac{H_t}{V + V_l} - \frac{V - V_l}{y} \qquad [15]$$

$$x = \frac{y}{2\frac{H_t}{V + V_l} - \frac{V - V_l}{y}} \qquad [16]$$

In FIG. 9 we give a set of **A** vs. **J** curves for 1.6 m/s rated speed and 0.1 m/s levelling speed for different stopping distances.

A computer program has been developed to draw these curve sets for the desired possible values of V, V_l and H_t. They may be examined on the screen or on a hard copy.

The second step constitutes the tolerance to allow for the errors arising from the different inaccuracies of the system. It should not exceed 1 s, which makes about 0.1 more metres of stopping distance. The third step's effect on stopping time and distance is negligible.

In some situations it may be advisable to forsake the symmetry of the deceleration curve since the rounding of the low speed knee has very little effect on the lengthening of the stopping distance and time. Practical results are very aproximately achieved supposing that both roundings have the value of the one at the high speed knee. For more precision the two step method described in section 8 may be used.

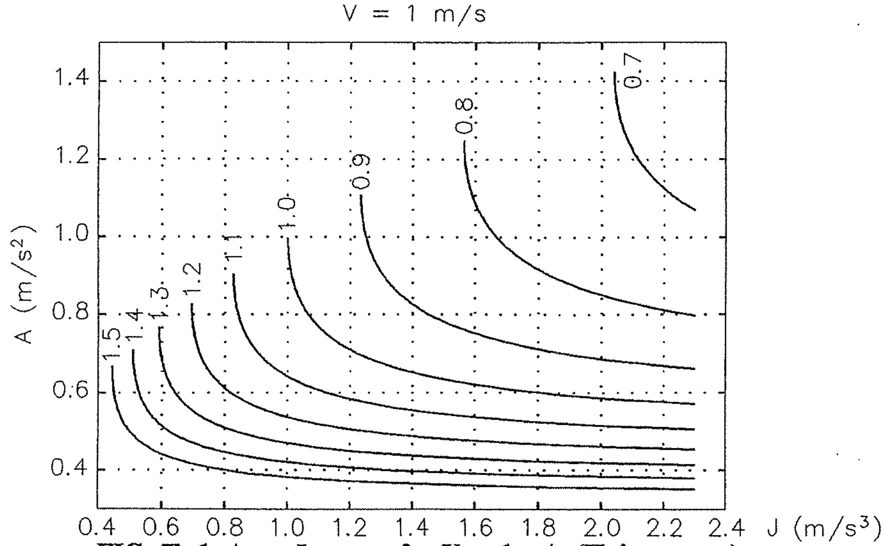


FIG. 7b-1: A vs. J curves for V = 1 m/s (H_t in metres).

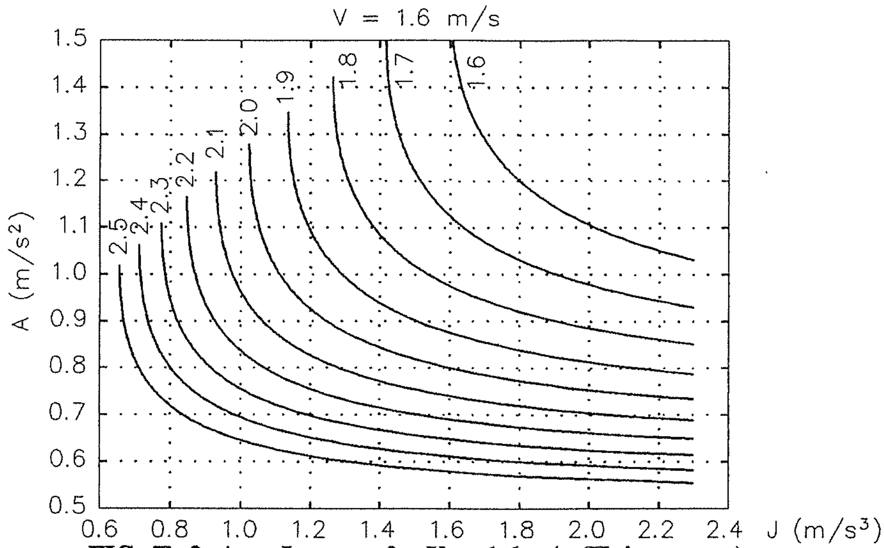


FIG. 7b-2: A vs J curves for V = 1.6 m/s (H_t in metres).

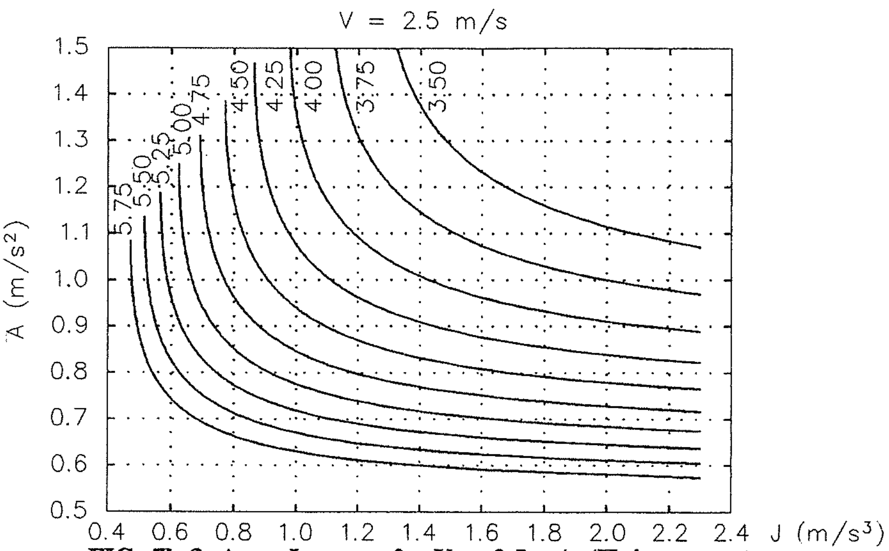


FIG: 7b-3: A vs. J curves for V = 2.5 m/s (H_t in metres).

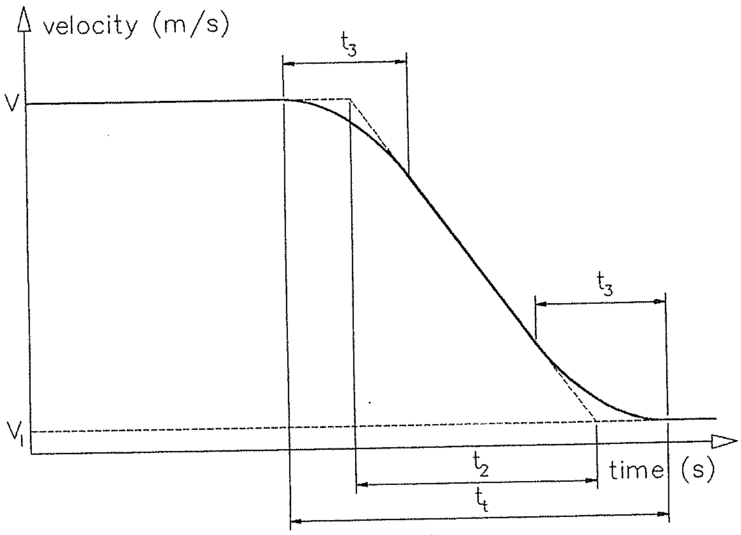


FIG. 8: Deceleration curve

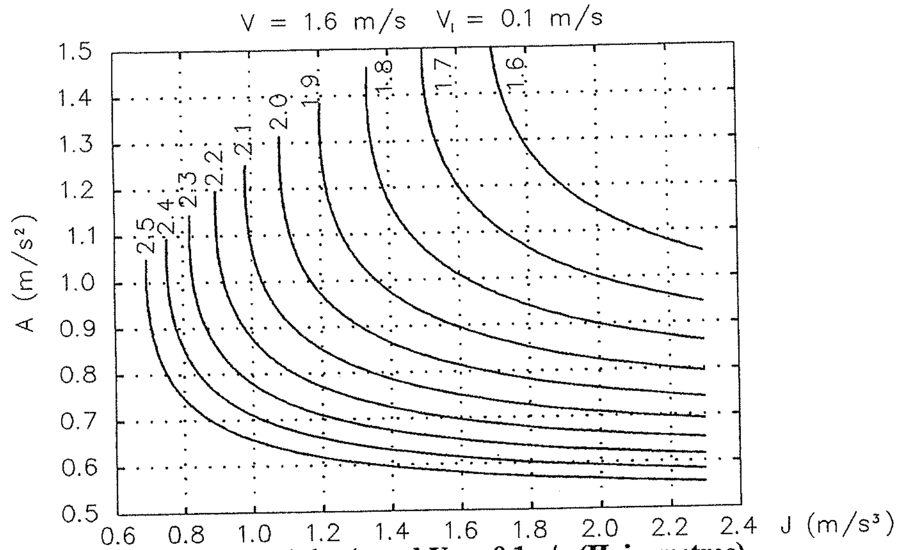


FIG. 9: A vs J curves for V = 1.6 m/s and V_l = 0.1m/s (H_t in metres)

8. TWO STEP METHOD OF CALCULATION OF DISTANCES.

This method consists of calculating, in a first step, the area defined by the tangents to the speed-time curve, which is an addition of triangles and rectangles; and then, making use of the table of Appendix 1, add or subtract the differences in the roundings of this curve. This method takes advantage of the fact that these differences are only a function of the maximum acceleration and maximum jerk considered in each of the roundings.

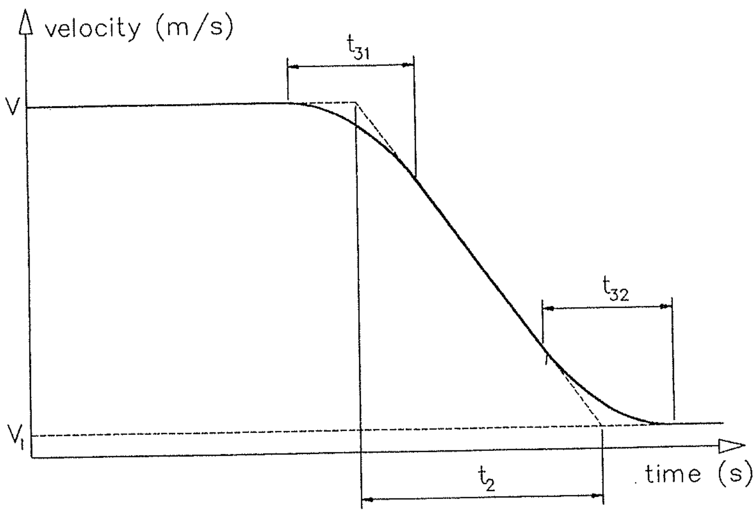


FIG. 10: Tangents and rounded parts

For the cases in which the distance between start and stop floors is too short to achieve rated speed, but not too short to reach maximum acceleration, the previously mentioned computer program may be used.

9. CONCLUDING

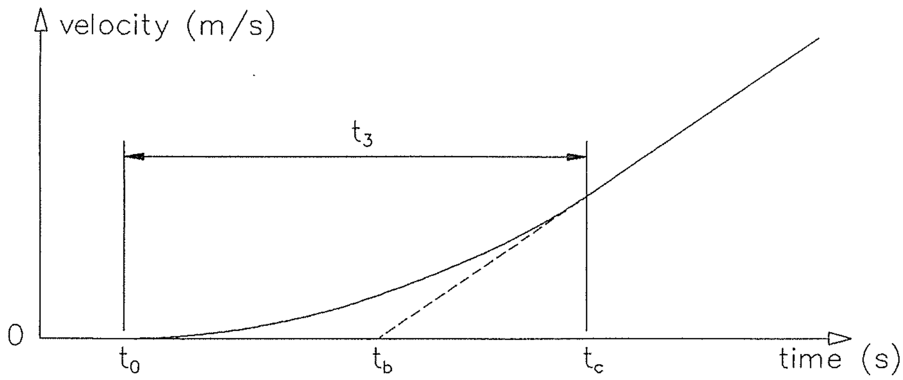
Again, it is worthy to point out that in our exposition no regard has been given to the duration of acceleration and jerk, nor the possible compound effect (on ride quality) of both together. This is hardly a realistic asumption, as has been shown by Dr. G. C. Barney (see ref. no. 2).

On the contrary, it is highly possible that by the use of non constant value jerks, for example, jerks that are first strong and then diminishing, the comfort may be improved without losing performance. At least the fact that maximum acceleration and maximum jerk do not coincide in time suggests that.

It is our opinion that when we reach a better understanding of discomforting effects of acceleration and jerk on human beings we will be able to advance significantly in providing our customers with the best possible comfort/performance value.

APPENDIX 1: Calculation of space differences between exact integrations and linear approximations in the roundings of the speed-time curve.

To calculate the area between a parabolically rounded speed-time curve and its tangents let us use the figure below.



Let us call:

- **H** the whole of the area below the curve between times t_0 and t_c.
- H_0 the area of the triangle formed under the second tangent between t_b and t_c.
- H_1 the area between the rounding and the tangents.
- **a(t)** the instantaneous value of the acceleration.
- **A** the value **a(t)** from t_c on, that is, its maximum.
- **v(t)** the instantaneous value of the speed.
- **h(t)** the instantaneous value of the space (distance).
- **J** the constant value of jerk from t_0 to t_c.

Then we have:

$$a(t) = \int_{t_0}^{t} J\,dt = J\cdot(t-t_0)$$

$$v(t) = \int_{t_0}^{t} a(t)\,dt = \int_{t_0}^{t} J\cdot(t-t_0)\,dt = \frac{J}{2}\cdot(t-t_0)^2$$

$$h(t) = \int_{t_0}^{t} v(t)\,dt = \int_{t_0}^{t} \frac{J}{2}\cdot(t-t_0)^2\,dt = \frac{J}{6}\cdot(t-t_0)^3$$

Consequently,

$$A = Jt_3$$

$$v_{t_c} = \frac{Jt_3^2}{2}$$

$$H = \frac{J}{6}t_3^3 = \frac{A^3}{6J^2}$$

Thus

$$\left.\begin{array}{l} H = \dfrac{A^3}{6J^2} \\ H_0 = \dfrac{1}{2}\dfrac{t_3}{2}\dfrac{At_3}{2} = \dfrac{At_3^2}{8} \end{array}\right\} \Rightarrow H_1 = H - H_0 = \frac{1}{24}\frac{A^3}{J^2}$$

From this we calculate the table below:

		A			
		0.8 m/s²	0.9 m/s²	1 m/s²	1.1 m/s²
J	1 m/s³	21 mm	30 mm	42 mm	55 mm
	1.25 m/s³	14 mm	19 mm	27 mm	35 mm
	1.50 m/s³	9 mm	14 mm	19 mm	25 mm
	1.75 m/s³	7 mm	10 mm	14 mm	18 mm
	2.00 m/s³	5 mm	8 mm	10 mm	14 mm

REFERENCES

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2. Barney G.C. "Let's look at lift dynamics again" Elevatori, July 1995: 79-81. (For a discussion on the effect of jerk).
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BIOGRAPHICAL DETAILS

Kepa Zubia is Dr. Industrial Engineer from the Industrial Engineering Superior Technical School of Bilbao. From 1963 to 1968 Dr. Zubia worked at General Eléctrica Española designing electronic equipment mostly to control and drive AC and DC motors. After GEE abandoned design in favor of mass production in 1968 he created Nafar Elektronika, a company dedicated mostly to the design of analog and digital controls. In 1992 he founded Kementsu, S.L., to design and produce electronic equipment for the elevator industry.