# Statistical Analysis of Modern Elevator Dispatch Hall Call Response Times.

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**Abstract**

Hall call response times of elevator dispatch systems have been statistically modeled by the Poisson process. This distribution has proved rather accurate when evaluating relay based dispatch algorithms. In the analysis of modern microprocessor based elevator dispatch algorithms, such as the Dynamic Sectoring Algorithm, the CGC Algorithm, or the Minimal Variance Algorithm it is clear that the statistical performance can no longer be modeled by the Poisson Process. This paper investigates other distributions that better describe the elevator dispatch system behavior as well as provides a hypothesis of why the Exponential Poisson Process provided a good fit in the past.

**Introduction**

Since the invention of automatic elevator systems, the statistics of hall call response times have been modeled solely by the Poisson Distribution (exponential). This model proved to be reasonably accurate for relay dispatch systems that did not use either minimal cost algorithms or intelligent divide and conquer algorithms. With the entrance of microprocessor based dispatch systems, more sophisticated algorithms have become available for implementation as well as better methods for simulation and evaluation of these algorithms. After evaluating these algorithms and comparing them to the exponential process, using a figure of merit evaluation system, it is now clear that the Poisson Process is no longer an accurate description of these systems.

I will compare the hall call response time distributions of several published algorithms, such as the Dynamic Sectoring Algorithm, and CGC Algorithm simulation results in [Barney](1), and the Minimal Variance Algorithm in [Halpern](3), that trades average call response time for increased variance performance. I will show that the histograms of the performance of these algorithms do not resemble the Poisson Distribution and that mathematically the Poisson Distribution provides a poor fit. In all of the above algorithms the response time histograms peak at some point in the distribution after the origin, unlike the exponential histogram that peaks at the origin. Furthermore, the variance is no longer equal to the mean response squared. This deviation in mean/variance relationship is a direct indicator that the statistics are no longer Poisson.

This paper will investigate other distributions that provide a better fit to the results of the algorithms discussed, such as the Gamma Distribution, Shifted Gamma Distribution and the Rayleigh Distribution.

**Background**

This paper relies heavily on the understanding of some very basic principles in probability theory. Let's review those principles and present some well known probability distributions and their respective histograms .

A probability density function denoted by P(x) shows the probability that an event will occur at a given value of x. A very familiar density function that appears rather frequently in the literature is the exponential distribution where

$$P(X) = e^{-x}$$

where x = r/R
r = Hall call response time.
R = Average hall call response time.

The exponential function is a special case of the Poisson distribution and is used to predict the probability of a hall call arrival as well as the arrival of an elevator in response to that hall call. The probability density function looks as follows.

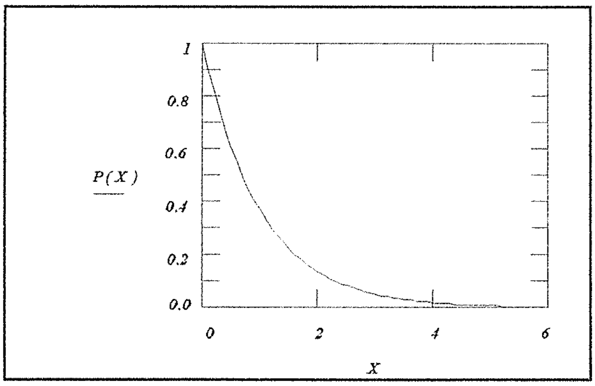



Figure 1.

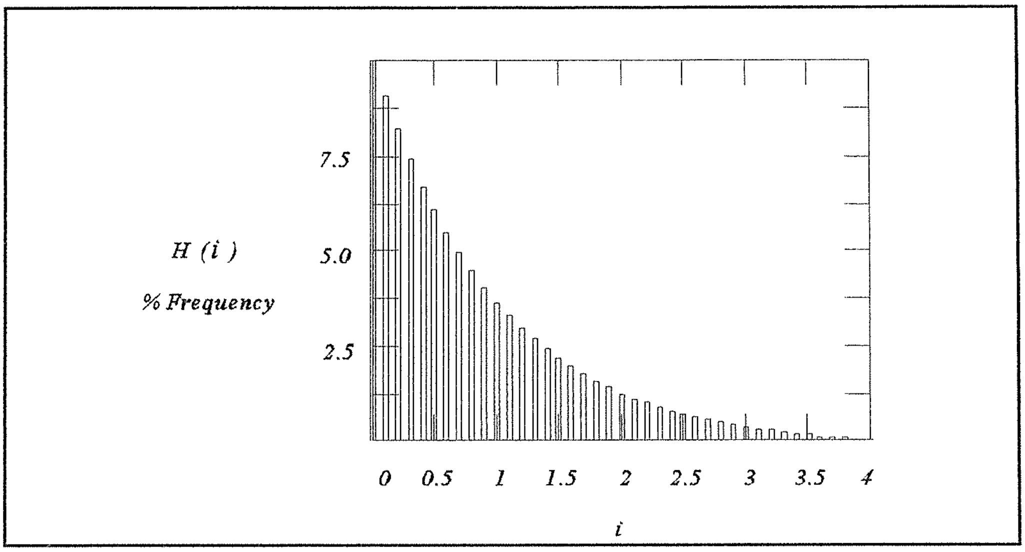
A second very important property of probability functions is that the area under the curve of the function must sum to one. This means that of all the possible combinations that x may take, if we add up all the probabilities associated with those values, they must sum to unity.

$$\int_0^\infty e^{-x} \, dx = 1.0$$

Similarly, if we look at the histograms that most elevator system analyzers produce, a similar result should be obtained by summing the percent of calls in each of the waiting time columns, and the sum should be 100%.

$$\sum h_i = 100\ \% \quad \textit{Each division of the Histogram}$$

Where $h_i$ is the percent or number of calls in each category of the histogram.




$h_i$ as a function of i for Poisson.
Figure 2.

**Normalization of Response Time Performance**

The performance distributions that will be presented will be all normalized with respect to the mean response time. All response times measured are performed in units of Average System Response Time (ASRT). This normalization allows us to look at systems with completely different average response times and compare relative performance [Barney](1). It also allows us to represent the variance performance factor directly from the histogram because in Poisson performance, the mean = variance = unity (1.0). A response time of 1.0 is equivalent to the average system response time which will vary from system to system. See the normalized response time of the exponential function plotted in Figure 2. H(i) is the percent of calls in each category divided by 100, and n is units of ASRT.

**Statistical Processes Considered**

The probability density functions considered are Exponential, Rayleigh, Gamma and Shifted Gamma, and will be compared to various algorithms to see how closely they match in performance. Below are the mathematical formula for those functions, and in Figures 4a-4d the corresponding histograms are displayed.

4 a. Exponential

$$P(X) = e^{-x}$$

4 b. Rayleigh Distribution

$$P(X) = x\, e^{-x}.$$

4 c. Gamma Distribution

$$P(x) = \left(\frac{x^2}{\sqrt{2\pi\sigma^2}}\right) e^{-\frac{x^2}{2\sigma^2}}$$

4 d. Shifted Gamma.

$$P(x) = \left(\frac{(x-a)^2}{\sqrt{2\pi\sigma^2}}\right) e^{-\frac{(x-a)^2}{2\sigma^2}}$$

x = r/R where r = the response time and R = the average response time.
a = shifting operator.

All the processes based on parameters are adjusted by a constant so that they integrate to one, (ie. normal functions).

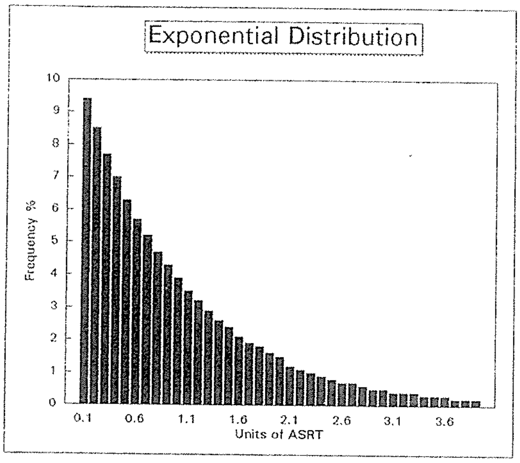
**Performance of Algorithms**

In Figure 5, graphs a-d, the performance histograms for four different algorithms are displayed. The first is a conventional relay system using a Closest Call Algorithm with actual, not simulated data [Halpern](2), followed the Dynamic Sectoring Algorithm simulation [Barney](1), CGC Algorithm simulation [Barney](1), and then a Minimal Variance Optimization Algorithm with actual data, not simulated [Halpern](3).

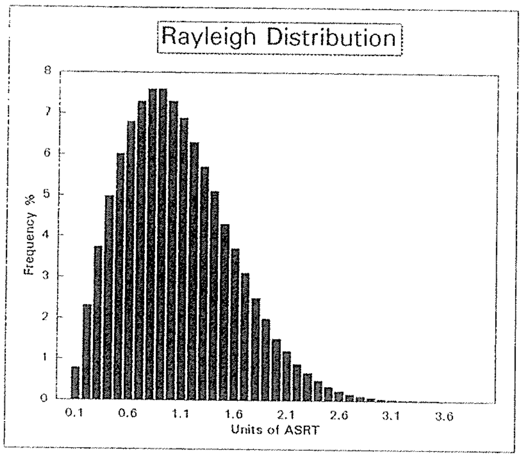
It is evident by observation of the figures themselves that the performance is not a pure exponential processes.

**Comparison Methods**

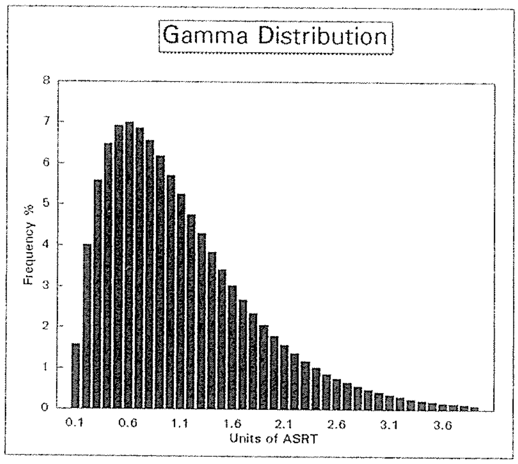
Each of the different histograms of the algorithms in figure 5 have been compared to the histograms of the statistical processes displayed in figure 4. A comparison figure of merit of the sum of squared error was calculated for each of the different algorithms comparing them to the different statistical processes. The method of calculation is shown in figure 6 and is basically the difference between the algorithm histogram and the statistical process histogram



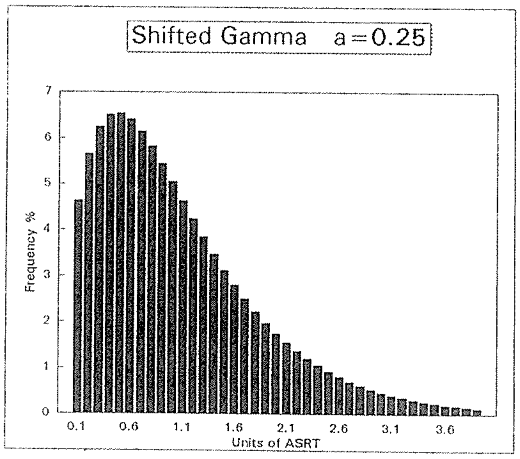

A.




B.

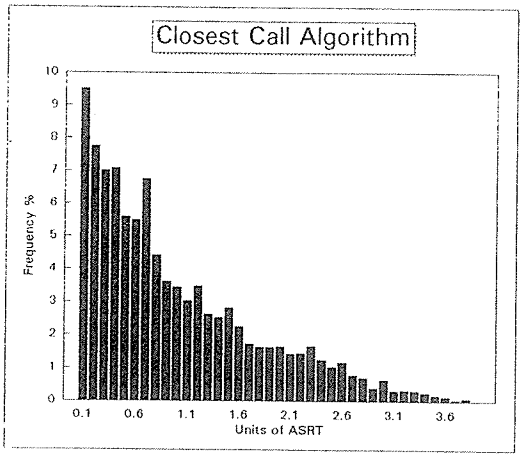


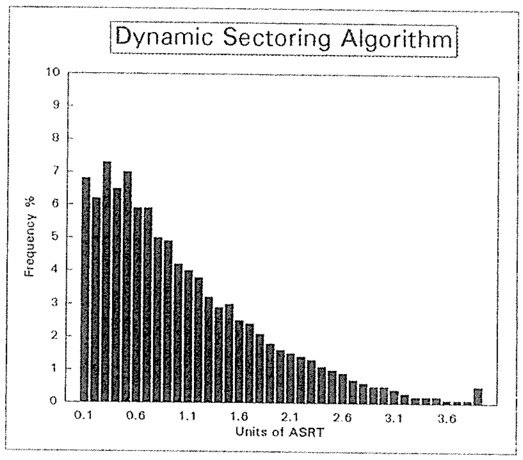

C.

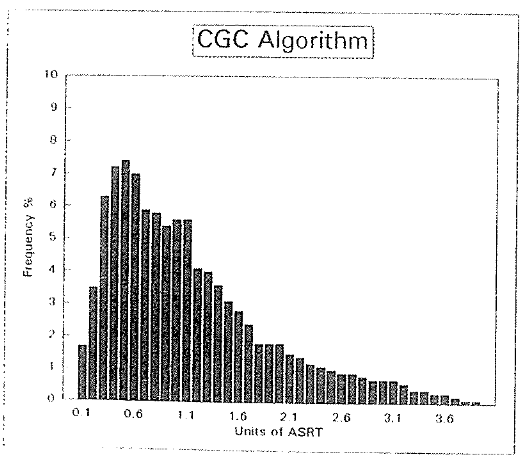



D.

Figure 4.



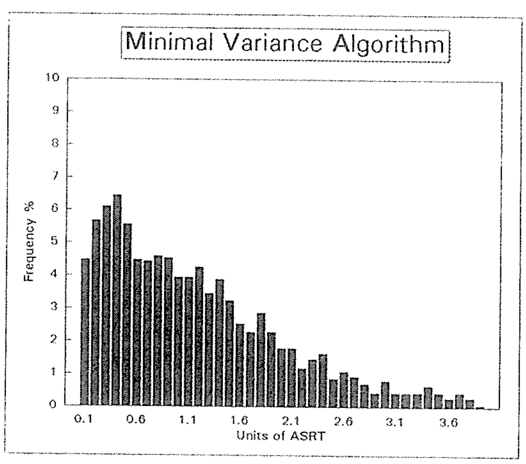




Figure 5.

in each discrete category squared and summed. The lower the figure of error squared, the closer the algorithm matches the statistical process. The error squared is a figure of merit proportional to the variance between the two compared functions. A table of the comparisons between the algorithms and the processes are shown in figure 7.

$$e^2 = \sum (h_i - h_s)^2$$

Sum of the error squared.
Figure 6.

| **Error Squared** | Exponential | Rayleigh | Gamma | Shifted Gamma |
|---|---|---|---|---|
| Closest Call | **6.2** | 46.6 | 107.2 | 52.2 |
| Barney DS | 104.4 | 48.7 | **5.58** | 8.64 |
| Barney CGC DP | 78.6 | 48.7 | 10.1 | **7.67** |
| Barney CGC | 16.6 | 49.1 | 43.7 | **10.1** |
| Minimum Var. | 45.3 | 40 | 35.6 | **13.6** |

Algorithm vs. Statistical Process ($e^2$)
Figure 7.

The best fit for each of the algorithms are bolded and underlined. A second best fit is underlined if it is significant. The added fifth algorithm CGC DP is the Barney CGC algorithm under down peak traffic conditions which was added because the data was available.

## Observations

Although the Gamma Distribution seems to be the perfect fit for the Barney DS Algorithm, and the Exponential process matches a relay system utilizing a Closest Call strategy, it is the Shifted Gamma Distribution that fits most of the processes. For the processes that the Shifted Gamma does not fit, it is a close second best fit with the exception of the Closest Call Algorithm. It is my belief that the Shifted Gamma can be made to fit all of the different systems performances curves by adjusting the shifting parameter "a", including the Closest Call system performance.

The hypothesis for this observation is as follows. Since the response time is typically calculated from the point a passenger requests service by initiating a hall call, until the point at which an elevator commits to a slow down and cancels the hall call, rather than measuring the response time until the point where the passenger receives actual service with the elevator cabin in a state with the doors open ready to receive the passenger, this disparity in measurement creates a dead time window between the two real world states, producing a shifting of the statistics toward the origin.

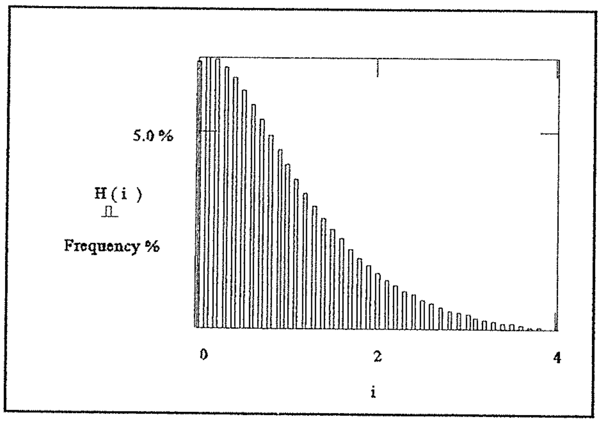
Secondly, since we are measuring hall call response time rather than actual response to the passenger waiting to leave the floor, the system has an aliasing problem with all passengers requesting service at the same time a car is making a coincident stop. The response to this passenger in the coincident time frame is compressed. The coincident window can easily be as large as ten to fifteen seconds. This coincident period causes an additional shifting of the statistics toward the origin.

If we were to measure actual response to the passenger, we would measure response time from the time the passenger requests service, (pushes the corridor call) until the time the passenger can enter the elevator cabin. If we measure these statistics we should observe actual Gamma or Erlang statistics. A zero length response time is virtually impossible because the elevator would have to be parked at the floor with the doors open when the passenger requests service. To receive real zero response time service a system would need an infinite number of elevators or something approaching infinite depending on the level of arriving passengers, and the number of floors served, similar to the results obtained from a simple queuing system.

## Analysis of Shifted Gamma Distribution

Since the Shifted Gamma Distribution fits nearly all the distributions with the exception of the Closest Call Algorithm, I would like to take closer look at the effects of the shifting operator "a". I will pick a value of "a" and regenerate the statistics for the Shifted Gamma following the formula given in figure 4d. I will then compare the results to the Closest Call Algorithm Distribution to see if I can obtain an acceptable fit.

If I pick a = 0.75 and calculate the error squared in comparison to the Closest Call Algorithm as was done before, then $e^2 =$ **10.2** for a = 0.75 . This comparison, although not a perfect match, is very close to the Closest Call statistics, and I believe one could fine tune "a" to get closer if necessary. Figure 8 shows the normalized histogram for the Shifted Gamma Distribution where a=0.75.




Shifted Gamma Histogram "a" = 0.75
Figure 8.

## Conclusion

Reasonably sound matches of statistical processes for each of the algorithms considered have been found. We can no longer assume that the statistical process that governs hall call response time is a pure exponential process (Poisson). Different algorithms behave in different ways, producing different statistics, and must be evaluated individually. However, for all the algorithms investigated within, the Shifted Gamma Distribution seems to be the best fit for all cases considered.

That the statistics of true system response time are Gamma and the reason that they appeared in the past to be exponential is due to the shifting of the statistics toward the origin by the selection of a performance measure that has an unmeasurable gap inherently built in. This gap is the time from the cancellation of a hall call request until the actual delivery of service to the passenger when the passenger enters the elevator cabin. This shifting phenomenon has produced statistics that show only the tail end of the Gamma statistics which appear to be exponential as shown in figure 8.

The reason that the Gamma statistics appear in the Barney simulations of the Dynamic Sectoring Algorithm and particularly in the CGC Algorithm, is that in the actual call allocation algorithm, there is a vehicle to shift potential allocations of very short response times to sub-optimal allocations closer to some fixed response time. This allocation strategy produces a partial shift of the response times away from the origin where hall calls are less likely to be responded to in the dead band time window, reproducing Gamma statistics.

## Acknowledgment

All Figures for the Barney DS and Barney CGC algorithms were taken from [Barney](1) with permission from Peter Peregrinus LTD.

## Bibliography

1. [Barney] **Elevator Traffic Analysis Design and Control, 2nd Edition** , G.C. Barney and S.M. dos Santos, Peter Peregrinus Ltd., London 1985

2. [Halpern] **Variance Analysis:** A new way of evaluating elevator dispatch systems. , Jon B. Halpern, Elevator World , Mobile, Alabama, September 1992

3. [Halpern] **Variance Analysis of Hall Call Response Time:** Trading mean hall call response time for increased variance performance. , Jon B. Halpern, Elevator Technology 5 , Proceedings of Elevcon '93, IAEE, Stockport, England, November 1993