ANALYSIS OF MOTION EQUATIONS OF ELEVATOR DRIVE SYSTEMS

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ABSTRACT

In this paper several forms of motion equations for worm and worm gear elevator drive systems are developed for varying loads and torques. Some conclusions are given as a basis for design and test of elevator drive systems.

SYMBOLS

a	acceleration of the car (m/s^2).
D	effective diameter of the driving sheave (m).
f	friction coefficient.
F_d	drive force (N).
F_l	load force (N).
i	reduction ratio of the gear box.
J_1	moment of inertia of the induction machine (kgm^2).
J_2	moment of inertia of the brake (kgm^2).
J_3	moment of inertia of the worm gear (kgm^2).
J_4	moment of inertia of the driving sheave (kgm^2).
J_5	moment of inertia of the defector sheave (kgm^2).
J_e	equivalent moment of inertia of the systems (kgm^2).
M	mass of the load in the car (kg).
M_1	mass of the car itself (kg).
M_2	mass of the counterweight (kg).
M_r	mass of the rated load (kg).
m	mass (kg).
n	rotational speed on the shaft of the induction machine (RPM).
n_1	rotational speed on the shaft of worm gear (or the driving sheave)(RPM).
P_1	output power of the induction machine (kw).
P_2	output power of the worm gear (kw).
P_r	power loss of the gear box when the worm drives the worm gear (kw).
P_r'	power loss of the gear box when the worm gear drives the worm (kw).
r	radius of the pulley (m).
T	output torque of the induction machine (Nm).
T_1	input torque of the gear box (Nm).
T_2	output torque of the gear box (Nm).
T_c	constant load torque of the system (Nm).
T_d	drive torque (Nm).
T_l	load torque (Nm).
T_{l1}	system load torque, positive power flow, static friction phase (Nm).
T_{l1}'	system load torque, positive power flow, kinetic friction phase (Nm).
T_{l2}	system load torque, negative power flow, static friction phase (Nm).
T_{l2}'	system load torque, negative power flow, kinetic friction phase (Nm).
T_r	system resisting torque, positive power flow, kinetic friction phase (Nm).
T_r'	system resisting torque, positive power flow, kinetic friction phase (Nm).

T_{rs} system resisting torque, positive power flow, static friction phase (Nm).
T_{rs}' system resisting torque, negative power flow, static friction phase (Nm).
v travel velocity of the car (m/s).
Z_1 number of threads on the worm.
Z_2 number of teeth on the worm gear.
η gear box efficiency when the worm drives the worm gear.
η' gear box efficiency when the worm gear drives the worm.
ω angular velocity on the shaft of the induction machine (rad/s).
ω_1 angular velocity on the shaft of the worm gear (rad/s).
ε angular acceleration on the shaft of the induction machine (rad/s^2).
ε_1 angular acceleration on the shaft of the worm gear (rad/s^2).
λ lead angle of the screw.
φ friction angle.

1. INTRODUCTION

An elevator exhibits two main characteristics, when considering the drive system: first, the drive system only provides upward and downward movement of a closed car, and secondly, through the processes of acceleration, constant speed, and deceleration accurate levelling is required at each floor. To design a high performance elevator, engineers are required to analyse in depth the motion equations controlling these characteristics.

2. MOTION MODEL AND SOME GENERAL CONSIDERATIONS

2.1 Motion Model

Most electric drive systems for elevators comprise a three phase induction machine, other electrical components and mechanical elements. The motion of the electric drive systems for an elevator can be easily described as a combination of translational and rotational motion.

Consider the system of *Fig.1*:

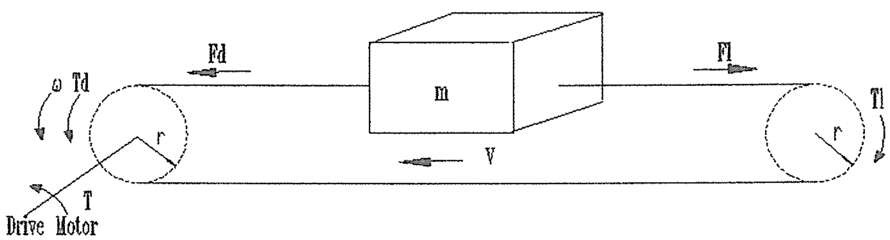

Figure 1 Rotary-to-linear motion model

where: $T_d = r\,F_d$, $T_l = r\,F_l$, and $v = r\,\omega$

Disregarding the moment of inertia of the pulley, the motion equations can be formulated (Leonhard, 1985; Kuo, 1982) from Newton's Law of Motion as follows:

$$T_d - T_l = r\frac{d(mv)}{dt} = mra = mr^2\frac{d\omega}{dt} = J_e\frac{d\omega}{dt} = J_e\varepsilon \qquad \text{... (1)}$$

2.2 Schematic Drawing and Operating Modes

The schematic drawing of the electric drive system with a worm and worm gear is shown in *Fig.2* with a hoist ropes ratio of *1:1*. The system consists mainly of eight elements shown in *Fig.2*.

Owing to the change of loads in the *Car 7* and the action of the *Counterweight 8* as well as the influence of the resisting torque of the system, the *Induction machine 1* will operate in all four-quadrants of the torque-speed diagram, as shown in *Fig.3*. From *Fig.2* and *Fig.3*, it can be seen that there are two possible power directions:

(A) Machine to Brake to Worm and Worm Gear to Driving Sheave to Hoist Ropes to Car and Counterweight (defined as *positive power flow direction*).

(B) Car and Counterweight to Hoist Ropes to Driving Sheave to Worm and Worm Gear to Brake to Machine (defined as *negative power flow direction*).

Therefore, the three phase induction machine used in an elevator can operate either as *an induction motor* which absorbs electric power, or as *an induction generator* in which the rotor is driven by the potential energy load torque beyond the synchronous speed such that the induction machine begins to deliver electric power.

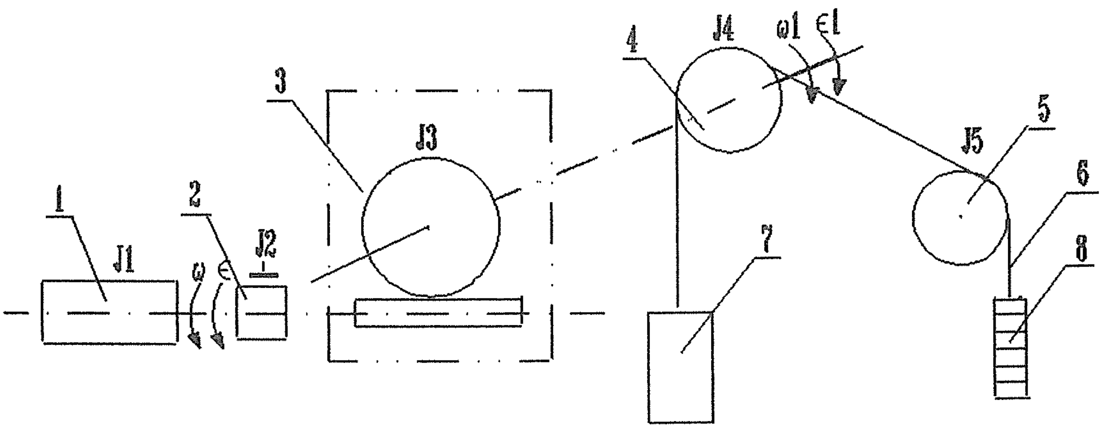

Figure 2 Schematic drawing of the system
1. Three phase induction machine, 2. Brake, 3. Worm and worm gear, 4. Driving sheave, 6. Hoist ropes, 7. Car, 8. Counterweight.

2.3 Equivalent Moment of Inertia of the System

Considering *Fig.2*, the equivalent moment of inertia of the system that the induction machine sees is evaluated as follows:

$$J_e = (J_1 + J_2) + \left(\frac{J_3 + J_4 + J_5}{i^2}\right) + (2M_1 + M + KM_r)\frac{D^2}{4i^2} \qquad \text{... (2)}$$

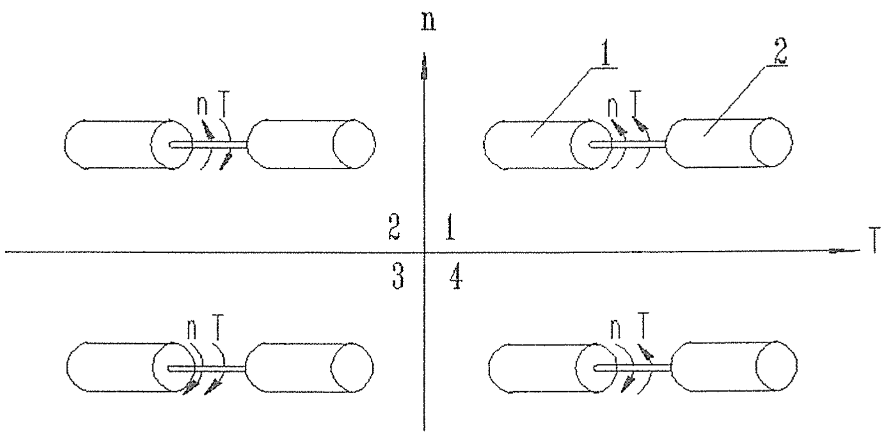

Figure 3 Operating modes of the system
1. Three phase induction machine, 2. Equivalent load of the system

The reduction ratio of the gear box is given by:

$$i = \frac{Z_2}{Z_1} = \frac{n}{n_1} = \frac{\omega}{\omega_1} = \frac{\varepsilon}{\varepsilon_1} \qquad \text{... (3)}$$

The balance coefficient *(K)* is related by:

$$K = \frac{M_2 - M_1}{M_r}, \; i.e. \; M_2 = M_1 + KM_r \qquad \text{... (4)}$$

From Equation *(2)*, the equivalent moment of inertia of the system mainly consists of the following three parts:

1. J_1+J_2: the sum of the moments of inertia of the induction machine and the brake;
2. $(J_3+J_4+J_5)/i^2$: the sum of the equivalent moment of inertia of the worm gear, the driving sheave and the deflector sheave that the induction machine sees;
3. $(2M_1+M+KM_r)\,D^2/4i^2$: the sum of the equivalent moment of inertia of the car and the counterweight that the induction machine sees;

Note: (J_1+J_2) is usually the most significant term in J_e. Also J_e varies slightly with the load in the car.

2.4 The Gear Box Efficiency

In general, the gear box efficiency is the same as that of a screw and nut (*Hindhede, 1983*). As for gear box used in an elevator, it is possible that the worm drives the worm gear or vice versa. As a result, the efficiency equation has two different forms.

Form A. When the worm drives the worm gear, the efficiency equation is given (*Thomas, 1971; Hundhede, 1983*) by:

$$\eta = (0.95 \sim 0.96)\,\frac{\tan\lambda}{\tan(\lambda+\varphi)} \qquad \text{... (5)}$$

where: φ is equal to arctan of the friction coefficient f.

Form B. When the worm gear drives the worm, the corresponding expression for efficiency is:

$$\eta' = (0.95 \sim 0.96)\,\frac{\tan(\lambda-\varphi)}{\tan\lambda} \qquad \text{... (6)}$$

The *Efficiency Equation (5)* and *(6)* show that if the physical parameters of the gear box have been determined, the efficiency equations when the worm drives the worm gear, or vice versa, can be obtained.

2.5 Resisting Torques of the System

As shown in *Fig.2*, the resisting torques of the system are mainly composed of following four parts:

Part A. Resisting torques of friction, which includes:

- *(a)* the resisting torque of friction between the worm and worm gear;
- *(b)* the resisting torque of friction between the car guide and the guide rail;
- *(c)* the resisting torque of friction between the counterweight guide and the guide rail;
- *(d)* the resisting torque of friction between the axial diameter in the driving sheave and in the deflector sheave.

Part B. Resisting torques of vibration produced by the vibration of the car and the counterweight during the period of travelling.

Part C. Resisting torque of elastic deformation resulted from the elastic deformation of the hoist ropes.

Part D. Pneumatic resisting torque caused by the air resistance of the car and the counterweight during the phase of travelling in the well.

For convenience in analysis, the resisting torques of the system are discussed under two circumstances: the worm drives the worm gear and vice versa. Assume that:

T_r is the resisting torque that the induction machine sees when the worm drives the worm gear,
T_r' is the resisting torque that the induction machine sees when the worm gear drives the worm.

T_r and T_r' are discussed separately in the theoretical evaluation method and the experimental investigations in the following:

2.5.1 Theoretical Evaluation Method

As far as the medium speed ($v \leq 2m/s$) elevator with the gear box is concerned, the resisting torque of friction of the system is the key parameter because the other resisting torques are much smaller and usually neglected. In addition, the total resisting torque of friction of the system is mainly dependent on the resisting torque of friction between the worm and the worm gear, and the other sources of friction can be neglected.

When the worm drives the worm gear, the efficiency η is written:

$$\eta = \frac{P_2}{P_1 + P_2} = \frac{T_2}{T_2 + T_r i} \qquad \text{... (7)}$$

From the above efficiency expression, the resisting torque T_r is derived in the following form:

$$T_r = \frac{(1 - \eta)\, T_2}{i\eta} = \frac{(1 - \eta)\, i\eta T}{i\eta} = (1 - \eta)\, T \qquad \text{... (8)}$$

From the *Equation (5)*, the resisting torque in the kinetic friction phase can be obtained. Furthermore, the resisting torque during the static friction phase T_{rs} can be also obtained as:

$$T_{rs} \cong 2T_r = 2(1 - \eta)\, T \qquad \text{... (9)}$$

When the worm gear drives the worm, the *Efficiency Expression* η' is similarly given as:

$$\eta' = \frac{P_1}{P_1 + P_r'} = \frac{T_1}{T_1 + T_r'} \qquad \text{... (10)}$$

The definition of P_1 and P_2 is the same as those for the efficiency expression.
Thus, we have:

$$T_r' = \frac{(1 - \eta')\, T_1}{\eta'} = \frac{(1 - \eta')T}{\eta'} \qquad \text{... (11)}$$

The resisting torque in the static friction phase is evaluated as the following form:

$$T_{rs}' \cong 2T_r' = \frac{2(1 - \eta')T}{\eta'} \qquad \text{... (12)}$$

2.5.2 Experimental Investigations

It is necessary to note that the resisting torques of the system are different in various phases of travel and for different types of elevators. For example, at the moment of starting of the elevator, the resisting torque is principally determined by the static friction. Once the car has travelled a short distance in the well, the resisting torque is mainly decided by the kinetic friction. The values of resisting torque for a system are dependent on the quality of installation of the elevator. Therefore, after the resisting torques of the system have been obtained by the Theoretical Evaluation Method, it is required to verify and modify them by an experimental investigation in a laboratory, in an elevator tower or on-site.

3. ANALYSIS OF THE MOTION EQUATIONS OF THE SYSTEM

Each phase is analysed for two conditions (*A*) and (*B*).

3.1 Acceleration for Upward Movement

Condition (A) When *(M-KM_r)* is larger than or equal to *0*, the three phase induction machine must operate as an induction motor in order to accelerate the car in the well. In this case, the direction of power flow of the system is positive. At this time, the acceleration a of the car travel is positive, and

$$a = \frac{D}{2}\varepsilon = \frac{D}{2}\frac{d\omega}{dt} \quad \text{... (13)}$$

Accordingly, the motion equation of the system shown in *Fig.2* is derived as the following:

$$T - [\frac{D}{2i}(M - KM_r) + T_{rs}] = J_e \frac{d\omega}{dt} \quad \text{... (14)}$$

That is:

$$T - T_{l1} = J_e \frac{d\omega}{dt} \quad \text{... (15)}$$

Where:

$$T_{l1} = T_c + T_{rs} \quad \text{... (16)}$$

$$T_c = \frac{D}{2i}(M - KM_r) \quad \text{... (17)}$$

It is necessary to notice that as soon as the car has travelled a short distance in the hoistway in the scheduled travel direction, the resisting torque of the static friction T_{rs} is replaced by that of kinetic friction T_r and the system begins a normal starting movement. At this moment, the motion equation of the system becomes:

$$T - (T_c + T_r) = J_e \frac{d\omega}{dt} \quad \text{... (18)}$$

$$T - T_{l1}' = J_e \frac{d\omega}{dt} \quad \text{... (19)}$$

Where:

$$T_{l1}' = T_c + T_r \quad \text{... (20)}$$

In fact, when (M - KM_r) is equal to *0*, the *Equation (19)* can be simplified as:

$$T - T_r = J_e' \frac{d\omega}{dt} \quad \text{... (21)}$$

Where:

$$J_e' = J_1 + J_2 + \frac{J_3 + J_4 + J_5}{i^2} + \frac{D^2}{2i^2}(M_1 + KM_r) \quad \text{... (22)}$$

Equation (21) illustrates that the travel state for motor when $(M - KM_r)$ is equal to 0 is an ideal travel state for an elevator. Under the circumstance, as soon as the motor balances the torque T_r it can make the car accelerate upwards. That is, the motor only absorbs a quite small electric power.

Condition (B) When $(M - KM_r)$ is smaller than *0* and $-D/2i\ (M - KM_r)$ is larger than T_{rs}' before the motor starts, the potential energy load torque drives the system, i.e. the motor operates as an induction generator in which the rotor is driven by the potential energy load torque. The negative electromagnetic torque T plays a braking role in the system, in order to avoid the car falling freely or to avoid the car overspeeding. At this moment, the power flow direction is negative and the motion equation is given as the following:

$$T - [\frac{D}{2i}(M - KM_r) + T_{rs}'] = J_e \frac{d\omega}{dt} \quad ... (23)$$

That is:

$$T - T_{l2} = J_e \frac{d\omega}{dt} \quad ... (24)$$

Where:

$$T_{l2} = T_c + T_{rs}' \quad ... (25)$$

Where: T_c is calculated from the *Equation (17)*, but its value is negative. Note that after T_{rs}' is overcome by the system, T_{rs}' is replaced by T_r' and the motion equation of the system becomes:

$$T - [\frac{D}{2i}(M - KM_r) + T_r'] = J_e \frac{d\omega}{dt} \quad ... (26)$$

That is

$$T - T_{l2}' = J_e \frac{d\omega}{dt} \quad ... (27)$$

Where:

$$T_{l2}' = T_c + T_r' \quad ... (28)$$

3.2 Constant Speed for Upward Movement

Condition (A) When $(M - KM_r)$ is larger than or equal to *0*, the motion equation of the system is simplified from the *Equation (19)* as follows:

$$T = T_c + T_r \quad ... (29)$$

Condition (B) When $(M - KM_r)$ is smaller than *0*, and $-D/2i\ (M - KM_r)$ is larger than T_r' then the motion equation is simplified from the *Equation (27)* as follows:

$$T = T_c + T_r' \quad ... (30)$$

3.3 Deceleration for Upward Movement

Condition (A) When $(M - KM_r)$ is larger than or equal to *0*, the motor will drive the worm and the motion equation is the same form as the *Equation (19)*.

Condition (B) When $(M - KM_r)$ is smaller than 0, the machine will be driven by the potential energy load torque and the motion equation is the same form as the *Equation (27)*.

3.4 Acceleration for Downward Movement
Condition (A) When $(M - KM_r)$ is smaller than or equal to 0, or when $(M - KM_r)$ is larger than 0, and $D/2i(M - KM_r)$ is larger than T_{rs}, the machine will operate as a motor and the motion equation is the *Equation (15)*. As soon as the car has travelled a short distance, the motion equation becomes the *Equation (19)*.
Condition (B) When $(M - KM_r)$ is larger than 0 and when $D/2i\ (M - KM_r)$ is larger than T_{rs}', the machine will operate as a generator and the motion equation is the same as the *Equation (24)*. After the car has travelled a short distance, the motion equation becomes *Equation (27)*.

3.5 Constant Speed for Downward Movement
Condition (A) When $(M - KM_r)$ is smaller than or equal to 0, or when $(M - KM_r)$ is larger than 0, and $D/2i\ (M - KM_r)$ is larger than T_r, the motion equation is the same as the *Equation (29)*.
Condition (B) When $(M - KM_r)$ is larger than 0, and $D/2i\ (M - KM_r)$ is larger than T_r' the motion equation is the same as the *Equation (30)*.

3.6 Deceleration for Downward Movement
Condition (A) When $(M - KM_r)$ is smaller than or equal to 0, or when $(M - KM_r)$ is larger than 0, and $D/2i\ (M - KM_r)$ is smaller than T_r, the motion equation is the same as the *Equation (19)*.
Condition (B) When $(M - KM_r)$ is larger than 0, and $D/2i\ (M - KM_r)$ is larger than T_r', the motion equation is the same as the *Equation (27)*.

4 CONCLUSIONS AND DISCUSSION

A. There are several forms of motion equations for an elevator electric drive system with worm and worm gear under varying load states and resisting torques compared to a conventional electric drive system which often has only one form of motion equation. These motion equations can be used to design or test the starting, constant speed and braking phases of an elevator electric drive system.

B. Before starting, the initial operational state of the three phase induction machine has been determined by the load in the car and the resisting torques of the system, which have been influencing the performance of the system through the full operational phase. For this reason, it is necessary to set up a control circuit to detect varying load directly (e.g. *a load detection circuit*) or a circuit to measure varying load indirectly (e.g. *a current feedback circuit*) and adjust the electric drive system according to the change of the load in the car and the value of the resisting torques of the system.

C. The electric machine used in an elevator is able to operate either as induction motor or as an induction generator. In an ideal state, when $(M - KM_r)$ is equal to 0, the machine only absorbs quite a small power.

D. Due to the different values of the resisting torque under different operational states of the system and at different operational moments, it is necessary not only to analyse and control the starting transient phase and braking transient phase respectively, but also to study them at different operational moments.

E. The operational efficiency of the system is not high due to the large friction loss (large resisting torque). Furthermore, the efficiency expressions have two different forms, when the worm drives the worm gear and vice versa. Therefore, when the system is designed, two forms of efficiency expressions must be taken into account and harmonized with each other in order to improve the operational efficiency of the system.

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