THE DEVIATED BENDING

The state of stress of elevator guide rails and their elastic deformation based on the change in the load distribution on the car floor.

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Foreword

The Italian regulation related to the construction, installation and operation of elevators prescribes that the guide rails must support the horizontal push transmitted by the car normally hanging, no matter what the position of its travel is, with a load which is the same as its capacity, evenly spread on any half of its floor.
Within the same article the regulation provides that the elastic deformation arrow must not be higher than seven millimeters.
As a hypothesis of constraint it provides that each part of guide must be considered simply leaning between two consecutive anchorages.
In the past, when the great majority of elevators used to be electric traction type, the checking of the above qualifications was done assuming simplifying hypothesis on the distribution of the load in the car (F 1 a & b), with the conviction, affirmed but not proved, that to such distribution did correspond the maximum stresses on the guide rails.
Through the coming and spreading on a large scale of hydraulic elevators with lateral suspension, it immediately appeared evident that the old practice of approximation was not reliable any longer.
To overcome this difficulty some solutions have been proposed which develop to a long series of little formulas which on turn get to some results which can easily be objected.
According to me the most correct way to behave consists of a detailed analysis which may branch to different analytic developments which take into account all implications involved, but finally synthesize into a few formulas taking to reasonably reliable results.

F. 1

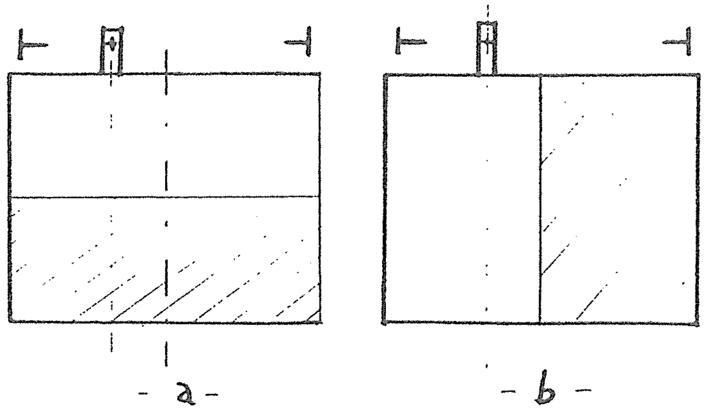

fig 1

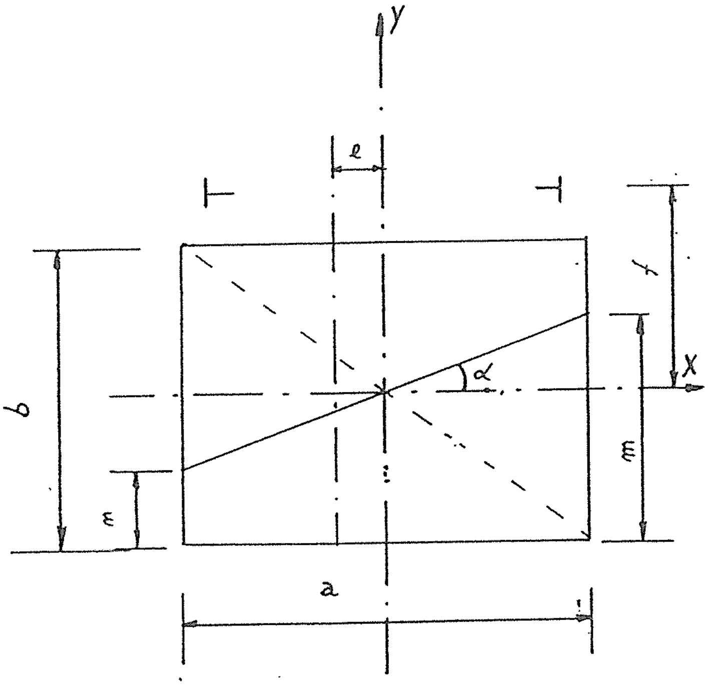

fig 2

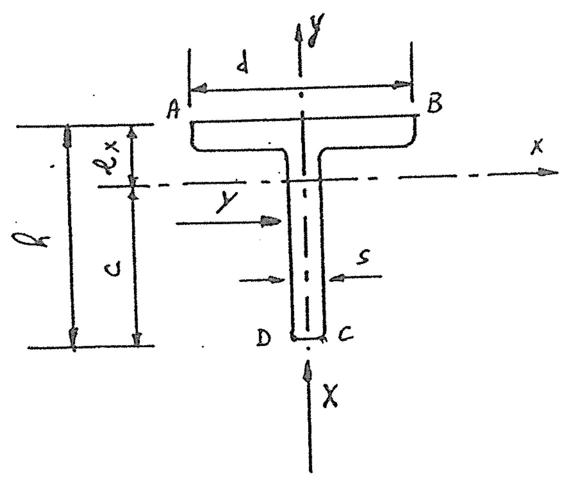

fig. 3

INTRODUCTION

In the majority of cases the floor of an elevator car has the shape of a regular quadrilateral and that is the case I will refer to.
However the criterion of calculation I propose is applied, without significant changes, to all cases where the floor of the car allows at least two axis of symmetry so that the whole of the straight lines dividing this floor into two parts equiextended constitute a sheaf of straight lines having thair pole to coincide with the symmetry centre.
In all other cases the methodology of investigation remains basically unchanged. Only the shape changes, which may articulate into far more complex formulas. Given that, to semplify the calculation development, it is good to make a distinction between the thrusts due to motionless loads (car frame load, operator load and every other accessory solid for any of such components) and those due to the traveling loads (loads differently distributed on one half of the car floor). The former ones, when the system geometry is determined, always remain unchanged and are easily calculable. The others change according to the distribution of the load. But because, as it was said, all the secants dividing the floor of the car into two equal parts, pass through the symmetry centre, each distribution is distinguished by one parameter only. Such a parameter may be the angle of inclination of the secant compared with one of the axis of symmetry of the car.
For example that parallel to the plate of the guides (F 2) which will be called α . Then assume a reference system coinciding with the two axis of symmetry X and y , imagine the load distributed on the farest half from the guide plate and express the coordinates of the barycentre according to the angle of inclination.

Barycentre Co-ordinates

Divide the α variation ground into two intervals. The first one changes from $\alpha = 0$ (secant parallel to the guide plate) to $\alpha = \mathrm{arctg}\,\frac{b}{a}$ (secant coinciding with the diagonal of the car.). The second one changes from $\alpha = \mathrm{arctg}\,\frac{b}{a}$ as far as $\alpha = \pi/2$ (secant perpendicular to the guide plate).
By applying the usual geometry rules it is found the following:

1) For

$$0 < \alpha < \mathrm{arctg}\,\frac{b}{a}$$

$$x_g = \frac{K \cdot a^2}{6\,b} \quad ; \quad y_g = \frac{K^2 \cdot a^2}{12 \cdot b} - \frac{b}{4}$$

having indicated the tag α with a K.
It is deduced that the geometric place for the barycentres is expressed by the following equation

$$yg = \frac{3 \cdot b}{a^2} \cdot x_g^2 - \frac{b}{4}$$

In particular:

$$\begin{cases} \text{For } \alpha = 0 , \quad K = 0 , \quad x_g = 0 , \quad y_g = -\frac{b}{4} \\ \text{For } \alpha = \operatorname{arctg}\frac{b}{a} , \quad K = \frac{b}{a} , \quad x_g = \frac{a}{6} , \quad y_g = -\frac{b}{6} \end{cases}$$

2)For $\operatorname{arctg}\frac{b}{a} < \alpha < \frac{\pi}{2}$

$$x_g = \frac{a}{4} - \frac{b^2}{12 K^2 \cdot a} , \quad y_g = -\frac{b^2}{6Ka}$$

In particular:

$$\begin{cases} \text{For } \alpha = \operatorname{arctg}\frac{b}{a} , \quad K = \frac{b}{a} , \quad x_g = \frac{1}{6} a , \quad y_g = -\frac{1}{6} b \\ \text{For } \alpha = \frac{\pi}{2} , \quad K = \infty , \quad x_g = \frac{a}{4} , \quad y_g = 0 \end{cases}$$

It is to be noted that the two arcs of parabola intersect on the point of co-ordinates ($\frac{a}{6}$, $-\frac{b}{6}$). This point belongs to the second diagonal.

The Critical Points

Figure 3 shows the normal section of a structural shape generally used as guide rail for an elevator car. The points of the section which are more stressed because of the parallel efforts of the two main axis of inertia are indicated with letters A, B, C, D.
But whereas on points A and C the effects of the two stresses of deviation are subtracted, on points D and B they are added up.
These two points will be called critical points.
Owing to the different alternate distances of the critical points from the main axis of intertia, as well as the different values of the thrusts, it is not possible to foresee by instinct which one of the two points is more stressed.
Therefore the investigation must be made on both critical points.
At a later stage it will be possible to make a comparative exam.

Critical Point B

The area of definition of stress σ_B on point B must be divided into two intervals which are to be examined separately.

1) Interval $0 < \alpha < \operatorname{arctg} \frac{b}{a}$

By indicating the components of the thrust on the guides parallel to the relevant axis of the symmetry with a X_1 and a Y_1 and remembering the expressions for the barycentre co-ordinates, it is found

$$\begin{cases} x_1 = \frac{Q}{h}\left(\frac{a^2}{6b}.k + d\right) \\ y_1 = \frac{Q}{2h}\left(-\frac{a^2}{12b}.k^2 + \frac{b}{4} + l\right) \end{cases}$$

Having indicated the total load in the car with a Q , and the vertical distance between the car shoes with a h.
The stress on point B is expressed by the following equation:

$$\sigma_B = \frac{Q\,l}{8\,h\,W_{yB}}\left[2\,\frac{W_{yB}}{W_{xB}}\left(\frac{k\,a^2}{6b} + d\right) - \frac{k^2 a^2}{12\,b} + \frac{b}{4} + l\right]$$

Where l indicates the vertical distance between two consecutive anchorages for the guide.

Giving $\beta = \frac{W_{yB}}{W_{xB}}$, this expression becomes

$$\sigma_B = \frac{Q\,l}{8h\,W_{yB}}\left[\frac{k\,a^2}{3\,b}\,\beta + 2\,d\,\beta - \frac{k^2 a^2}{12\,b} + \frac{b}{4} + l\right]$$

By deriving σ_B compared with K it is obtained:

$$\frac{\partial \sigma_B}{\partial k} = \frac{Q\,l}{8h\,W_{yB}}\left[\frac{a^2}{3b}\,\beta - \frac{a^2}{6b}\,k\right] = \frac{Q\,l\,a^2}{48\,h\,b\,W_{yB}}\left(2\beta - k\right)$$

Where one can see that $\frac{\partial \sigma_B}{\partial k} = 0$ for $k = k' = 2\beta = 2\,\frac{W_{yB}}{W_{xB}}$

Also it is to be noted that

$$\frac{\partial^2 \sigma_B}{\partial k^2} = -\frac{Q\,l\,a^2}{48\,h\,b\,W_{yB}} < 0$$ all the time.

That means that K is a peak-point for function σ_B . At this point it is necessary to make the following remarks:

a) As WyB is always other than zero, K' is always higher than zero and so is $\alpha' < \operatorname{arctg} k'$ all the time.
The peak-point will never be reached when the load is distributed on half of the front of the car.

b) For $k' < \frac{b}{a}$, namely $\alpha' < \operatorname{arctg} \frac{b}{a}$
the peak for σ_B will be obtained for $K=K'=2\beta$.

c) For $k' = \frac{b}{a}$ namely $\alpha' = \text{arctg} \frac{b}{a}$

the peak for σ_B will be reached when the load is distributed on half of the car area divided according to its diagonal

d) For $k' > \frac{b}{a}$ namely $\alpha'_{,} > \text{arctg} \frac{b}{a}$

the peak point is to be looked for in the second interval of definition.

2) Interval $\text{arctg} \frac{b}{a} < \alpha < \frac{\pi}{2}$

The stress on point B will always be expressed by:

$$\sigma_B = \frac{Q\,l}{8h\,W_{yB}} \left[\frac{1}{2} a\beta - \frac{1}{6} \frac{b^2}{a k^2} \beta + 2d\beta + \frac{b^2}{6ak} + e \right]$$

which derivative as compared with K is given by:

$$\frac{\partial \sigma_B}{\partial k} = \frac{Q\,l\,b^2}{48 h a W_{yB} k^3} \left(2\beta - k \right)$$

Once more the $\frac{\partial \sigma_B}{\partial k} = 0$ for per $k = k' = 2\beta = 2 \frac{W_{yB}}{W_{xB}}$

whilst as for the derivative number two

$$\frac{\partial^2 \sigma_B}{\partial k^2} = \frac{Q\,l\,b^2}{24 h a W_{yB} k^4} \left(-3\beta + k \right)$$

which is equal to zero for $k = k^* = 3\beta = 3 \frac{W_{yB}}{W_{xB}}$

The curve suffers a deviation just in this point.
To make it more clear how function σ_B changes, it may be made a short summary:

- for $k < 2\beta$, $\frac{\partial \sigma_B}{\partial k} > 0$, $\frac{\partial^2 \sigma_B}{\partial^2 h^2} < 0$

  the curve grows with its concavity looking downwards

- for $k = 2\beta$ $\frac{\partial \sigma_B}{\partial k} = 0$ $\frac{\partial^2 \sigma_B}{\partial^2 k^2} < 0$

  the curve reaches its peak

- for $2\beta < k < 3\beta$ $\frac{\partial \sigma_B}{\partial k} < 0$ $\frac{\partial^2 \sigma_B}{\partial^2 k^2} < 0$

  the curve decreases with its concavity looking downwards

- for $k = 3\beta$ $\frac{\partial \sigma_B}{\partial k} < 0$ $\frac{\partial^2 \sigma_B}{\partial k^2} = 0$

  the curve finds a flex-point

- for $k > 3\beta$ $\quad \frac{\partial \sigma_B}{\partial h} < 0 \quad \frac{\partial^2 \sigma_B}{\partial k^2} > 0$

the curve decreases with its concavity looking upwards.

It is possible now to get down to examine the different cases possible.

a) for $k = k' = 2\beta < \frac{b}{a}$

the peak point is to be looked for in the first space of definition;

b) for $k = k' = 2\beta = \frac{b}{a}$

the peak for σ_B is obtained when the load is distributed on half of the car area divided according to its diagonal;

c) for $k = k' = 2\beta > \frac{b}{a}$

the peak for σ_B is reached by $K = K' = 2\beta$;

d) as WxB is always other than zero

$k' = 2\beta = 2\,\frac{W_{yB}}{W_{xB}} < \infty$ all the time

and therefore $\alpha' < \frac{\pi}{2}$ all the time.

The distribution of the load on the lateral half of the car will never correspond to a peak of σ_B

Critical Point D

The equations expressing the stress σ_D according to K are formally identical to those relating to point B. The only difference consists of the different values of the resistance modules to bending.

The peak point for σ_D does not coincide with the peak point for σ_B.

1) Interval $0 < \alpha < \mathrm{arctg}\,\frac{b}{a}$

To make calculation easier it is assumed the compression stress as positive. Therefore the generical expression of σ_D is given by the following equation

$$\sigma_D = \frac{Q\,l}{8\,h\,W_{yD}}\left[\frac{k a^2}{3b}\gamma + 2d\gamma - \frac{k^2 a^2}{12\,b} + \frac{b}{4} + e\right]$$

having assumed $\gamma = \frac{W_{yD}}{W_{xD}}$ moreover

$$\frac{\partial \sigma_D}{\partial k} = \frac{Q\,l\,a^2}{48\,h\,b\,W_{yD}}\left(2\gamma - k\right)$$

which is equal to zero for $k = k^* = 2\gamma = 2\,\frac{W_{yD}}{W_{xD}}$

and finally $\frac{\delta^2 \sigma_D}{\delta k^2} = - \frac{Q\, l\, a^2}{48\, h\, b\, W y_D} < 0$

Therefore K" is the peak point for function σ_D .
In addition, the same remarks made for σ_B apply to this too.

a) The peak point will never be reached with load distributed on the front half of the car.

b) For $K = K'' < \frac{b}{a}$

The peak for σ_D will be obtained for $K = K'' = 2\gamma$

c) For $K = K'' = \frac{b}{a} = 2\gamma$

(load distributed on half of the car divided by its diagonal).

d) For $K = K'' > \frac{b}{a}$

The peak point is to be looked for in the second space of definition.

2) Interval $\quad \operatorname{arctg} \frac{b}{a} < \alpha < \frac{\pi}{2}$

$$\sigma_D = \frac{Q\, l}{8\, h\, W y_D} \left(\frac{1}{2}\, a\gamma - \frac{1}{6}\, \frac{b^2}{a k} \gamma + 2 d\gamma + \frac{b^2}{6 a k} + e \right)$$

from which $\quad \frac{\delta \sigma}{\delta k} = \frac{Q\, l\, b^2}{48 h a W y_D k^3} \left(2\gamma - k \right)$

which is equal to zero for $K = K'' = 2\gamma = 2\, \frac{W y D}{W x D}$

where there is the peak point.

Besides $\quad \frac{\delta^2 \sigma_D}{\delta k^2} = \frac{Q\, l\, b^2}{24 h a W y_D k^4} \left(- 3\gamma + k \right)$

which is equal to zero for $K = K'' = 3\gamma = 3\, \frac{W y D}{W x D}$

which is the flex-point.

The process for curve σ_D is therefore totally similar to that for curve σ_B .
The different positive cases are the following:

a) for $K = K'' = 2\gamma = 2\, \frac{W y D}{W x D} < \frac{b}{a}$.

The peak point is to be looked for in the first space of definition.

b) for $K = K'' = 2\gamma = 2\,\frac{WyD}{WxD} = \frac{b}{a}$.

The peak for σ_D is obtained with a load distributed on the car divided according to its diagonal.

c) for $K = K'' = 2\gamma > \frac{b}{a}$

The peak for σ_D is obtained with $K = K'' = 2\gamma$.

d) as WxD ia always other than zero

$K'' = 2\gamma = 2\,\frac{WyD}{WxD} < \frac{\pi}{2}$ always

and therefore $\alpha'' = \text{arctg}\,K'' < \frac{\pi}{2}$ always.

The distribution of the load on the lateral half of the car area will never correspond to a peak of σ_D .

<u>Comparative analysis between the maximum stresses in critical points</u>

To make the research for the maximum stresses easier, it is advisable to prepare a synoptic table stating the geometric and mechanic features of the plates generally used as guide rails. In the table below it is shown the features of guide rails mostly used, having taken the relevant data from the catalogue of a well known manufacturer of guide rails for elevators.

TYPE	Jx cm^4	W_{xB} cm^3	W_{xD} cm^3	Jy cm^4	W_{yB} cm^3	W_{yD} cm^3	$2\frac{W_{yB}}{W_{xB}}$ 2β	$2\frac{W_{yD}}{W_{xD}}$ 2γ
607 60×60×7	25,90	15,70	6,10	12,80	4,30	36,57	0,55	11,99
708 70×70×8	47,50	23,75	9,60	23,20	6,63	58,00	0,56	12,08
809 80×80×9	79,00	33,62	14,11	39,00	9,50	86,67	0,57	12,28
975 75×90×16	100,86	37,92	15,91	53,22	11,82	66,53	0,62	8,36
125 82×125×16	151,49	62,34	26,25	161,47	25,84	201,84	0,83	15,38