The systems analysis and design of lifts (elevators): the models and assumptions appraised

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\textbf{Abstract.} With modern computer systems equipped with relevant software tools / programs to automate the calculations, the systems analysis and design of lifts appears to be relatively straightforward. However, thorough understanding of engineering principles and models applied is of paramount importance in conducting the system calculations. This is critical in correct understanding of the assumptions applied in the safety standard formulae and requirements. In this context the importance of application of design / structural integrity criteria associated with the worst-case scenario dynamic conditions, to achieve a system which complies with accepted safety code requirements, is discussed and appraised. The paper demonstrates this through practical examples involving traction-drive systems designed to operate across a range of system design parameters.

\section{INTRODUCTION}

The design of a lift system involves bringing individual components together to arrive in a complete vertical transportation system (VTS) which will comply with the requirements of safety standards and codes and will carry the required load at the required speed over the necessary travel \cite{1}.

In any manufacturing environment, the process of system design must start with a preliminary selection of equipment. In particular, in order to commence design calculations, there must be an initial selection of hoist machine and diverter/secondary pulley. Selection of the machine must, in the first place, treat the machine as a structural member and will be based on the specified rated load and rated speed, together with an estimate of car mass based on the manufacturer’s product range. Having selected a machine with its traction sheave and diverter pulley, compliance with the minimum 40:1 sheave/rope diameter ratio specified by EN81 will now determine a maximum permissible rope size.

The design process is based on system calculations which involve the application of suitable models with the calculations based on engineering principles. The complete VTS is a dynamic system with time-varying loading conditions and parameters.

The detail of the system calculation may well require a review of the initial equipment selection, since issues such as compensation and alterations to the balance factor may result in a total loading which exceeds the rating of the initially selected machine. This simply demonstrates that in common with the majority of engineering systems, design is an iterative process, both in detail and overall.

This paper demonstrates how simplified models are developed and applied in the system calculation.
2 SYSTEM DESIGN CALCULATION

The principles of engineering mechanics form the foundation for system analysis and design of a VTS. The first stage involves the system identification by developing a physical model. Engineering systems, such as VTS, are often very complex and certain simplifying assumptions must be made beforehand. It is expected that the simplified model will then represent the behaviour of the actual system reasonably well.

A simple model of a traction-drive lift system (without compensation) is presented in Fig. 1(a). The fundamental design calculation for the lift system involves the following parameters:

- an estimate of the frictional characteristics of the traction system,
- the rated load and speed,
- an estimate of the masses/inertias of the various components in the hoistway and,
- the maximum accelerations to be expected under normal and emergency conditions.

This model may involve a number of simplifying assumptions. For example, in the first instance it can be assumed that:

- the rail guides are perfectly rigid so that the car and the counterweight are constrained in the horizontal direction and can move freely in the vertical direction only;
- vibration effects of the ropes/car, counterweight can be neglected.

In the fundamental analysis and system design calculations these assumptions and parameters are used to calculate the nature and mass of compensation means (if any) required to avoid slippage between the ropes and the driving sheave under defined conditions. For safety reasons, the systems calculation also seeks to guarantee that under some circumstances slippage between ropes and traction sheave must occur.

Consider a schematic diagram of a simplified model of the lift system without compensation means shown in Fig. 1(a). The diagram in Fig. 1(b) shows the suspension rope tensions $T_{\text{car}}$ and $T_{\text{cwt}}$ at the traction sheave at the car side and counterweight side, respectively. The angle of wrap of the ropes on the sheave is denoted as $\alpha$. In considering the system calculation, the procedure prescribed in EN81-50 clause 5.11.2 is to be followed so that the following inequality formulae, that originate from the Eytelwein-Euler equation [1], are applied:

$$\frac{T_1}{T_2} < e^{\frac{\alpha}{2}}$$

(1)

for traction to be maintained during normal operation/car loading and emergency braking conditions, or

$$\frac{T_1}{T_2} > e^{\frac{\alpha}{2}}$$

(2)

for traction to be lost during car/counterweight resting on the buffers (stalled conditions), where $T_1$ and $T_2$ represent the greater and the lesser ($T_1 > T_2$) dynamic tensions in the suspension ropes at either side of the traction sheave (representing either $T_{\text{car}}$ or $T_{\text{cwt}}$ respectively, depending on the loading/position in the hoistway conditions). In inequalities in equations (1-2) $e = 2.718\ldots$ is the natural
logarithm base, and \( f \) is the friction factor which depends on the coefficient of friction \( (\mu) \) as well as on the geometry of the rope – sheave contact configuration.

The determination of traction requirements is the fundamental consideration in lift system calculation. As noted earlier, the maximum possible rope size is constrained by the sheave/pulley diameter(s) on the selected traction machine.

The first step is the selection of suspension rope size. In order to make an initial selection of the rope size and number of ropes, the minimum safety factor needs to be established by considering the procedure in EN81-50 clause 5.12. Consider a low speed lift installation with the fundamental system parameters shown in Table 1, where the constant \( g = 9.81 \text{ m/s}^2 \) represents the acceleration of gravity.

<table>
<thead>
<tr>
<th>Load ( Q ) [kg]</th>
<th>Car mass ( P ) [kg]</th>
<th>Travel ( H ) [m]</th>
<th>Traction sheave diameter ( D_t ) [m]</th>
<th>Diverter pulley diameter ( D_p ) [m]</th>
<th>Rated speed ( V ) [m/s]</th>
<th>Normal acc./decc. ( a ) [m/s²]</th>
<th>V-groove Angle ( \gamma ) [°(deg)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>800</td>
<td>1000</td>
<td>25</td>
<td>560</td>
<td>560</td>
<td>1</td>
<td>0.1g</td>
<td>40</td>
</tr>
</tbody>
</table>

The minimum required safety factor is determined by the formula given in EN81-50 clause 5.12.3 which can be re-written as

\[
S_f = 10^F
\]

\[
F = 2.6834 - \frac{8.8425 + \log_{10} \left( N_{\text{equiv}} \right) - 8.567 \log_{10} \left( \frac{D_t}{d_r} \right)}{1.8870 - 2.894 \log_{10} \left( \frac{D_p}{d_r} \right)}
\]  

(3)

where \( N_{\text{equiv}} \) is the equivalent number of pulleys, \( d_r \) denotes the diameter of the rope.

For the V-groove angle \( \gamma = 40^\circ \), the equivalent number of pulleys is determined as [4]

\[
N_{\text{equiv}} = N_{\text{equiv}(t)} + N_{\text{equiv}(p)}
\]

(4)

where \( N_{\text{equiv}(t)} = 10 \) and \( N_{\text{equiv}(p)} = 1 \) so that \( N_{\text{equiv}} = 11 \). Consider using a standard rope size, \( d_r = 13 \) mm, giving a sheave/rope diameter ratio 43.08. Using \( N_{\text{equiv}} = 11 \) and \( D_t/d_r = 43.08 \) in (3) the minimum safety factor is determined as \( S_f = 17.18 \).

On the other hand the safety factor of the suspension means is defined as the ratio between the minimum breaking load of one rope and the maximum force in this rope, when the car is stationary at the lowest landing, with its rated load [2].
The minimum breaking load for a 13 mm 8×19 S - FC [4] is $F_{bmin} = 80.2$ kN and the corresponding nominal mass per metre is approximately $m_{sr} = 0.569$ kg/m. The total length of the suspension ropes on the car side when the car is stationary at the lowest landing can be estimated as $L_{sr} = H + l_{head}$ where an additional length in the headroom is added to the travel height. The ‘applied’ safety factor is then calculated as

$$S'_f = \frac{n_{sr}F_{bmin}}{(P + Q + n_{sr}m_{sr}L_{sr})g}$$ \hspace{1cm} (5)

where $n_{sr}$ denotes the number of ropes. For compliance with EN81-50 clause 5.12.3 the following condition must be satisfied

$$S'_f \geq S_f$$

i.e.

$$\frac{n_{sr}F_{bmin}}{(P + Q + n_{sr}m_{sr}L_{sr})g} \geq S_f$$ \hspace{1cm} (6)

from which the number of ropes required to give compliance may be calculated. By using $n_{sr} = 4$ ropes and $l_{head} \approx 3.5$ m ($L_{sr} = 25$ m+3.5 m = 28.5 m in (5)) the applied safety factor $S'_f$ is
determined as 17.54 which would be just adequate for the installation, provided subsequent calculations do not require an excessive increase in the well masses (e.g. due to the application of compensation).

Next, the traction calculation should be carried out. In order to determine the critical traction ratio (defined as $e^{1/\alpha}$) one needs to apply appropriate values of the friction factor $f$. Consider the case of car loading and emergency braking and that the V-grooves have been submitted to a hardening process. In that case the following formula applies [3]:

$$f = \frac{\mu}{\sin \frac{\gamma}{2}}$$  \hspace{2cm} (7)

where the coefficient of friction is determined as

$$\mu = 0.1 - \text{for normal operation/loading conditions}$$

$$\mu = \frac{0.1}{1 + \frac{V}{10}} = 0.091 - \text{for emergency braking conditions}$$  \hspace{2cm} (8)

The friction factor is then calculated as 0.2924 and 0.2658, for the loading condition and emergency braking condition, respectively. To determine the angle of wrap $\alpha$ let’s consider the diagram shown in Fig. 2.

![Figure 2: Traction sheave and diverter pulley geometry](image)

If the diameters of the traction shave and the diverter pulley are assumed to be the same $D_t = D_p = D$ with the distance between the rope centres denoted as $\Delta$, the angle of wrap is determined in terms of the vertical separation, $h$, of the sheave – diverter pulley and $\Delta$ as $\alpha = \pi - \tan^{-1}\left(\frac{\Delta - D}{h}\right)$ [1]. Consider that the rope centre distance is provided in the installation specification as $\Delta = 1150$ mm whilst the vertical separation is $h = 700$ mm. The angle of wrap is then determined as $\alpha = 139.87^\circ$. The critical traction ratios are then calculated as 2.04 and 1.91 for the loading condition and emergency braking condition, respectively.

According to the code requirements [2], the applied static traction ratio should then be evaluated for the worst-case depending on the position of the car in the well with 125 % of the rated load. Consider the static applied traction ratio with the car at the bottom landing. Assuming the balance $B = 0.45$, the tensile forces in the ropes at the traction sheave end/ diverter pulley end are determined as follows.
The corresponding applied traction ratio is then determined as \( \frac{T_1}{T_2} = 1.51 < 2.04 \). Thus, it is evident that traction in this scenario will be maintained.

In the case of emergency braking condition, the applied dynamic ratio is be evaluated for the worst-case depending on the position of the car in the well and the load conditions (empty, or with rated load). The calculation in the case of emergency stop at the deceleration rate of \( a_b = a = 1 \text{ m/s}^2 \) near the bottom landing whilst a full car is travelling downwards is given below.

\[
\begin{align*}
T_1 &= (P + Q + n_m m_s L_s) \left( g + a_b \right) = 20.159 \text{ kN} \\
T_2 &= (P + BQ + n_m m_s l_{head}) \left( g - a_b \right) = 12.052 \text{ kN}
\end{align*}
\]  

The corresponding applied traction ratio is then determined as \( \frac{T_1}{T_2} = 1.67 < 1.91 \) so that it is evident that traction will be ensured in this case as well.

### 3 Dynamic Traction Under Adverse Dynamic Conditions (Resonance Vibration)

Consider the longitudina elasticity (stiffness) \( EA \) of the suspension rope, where \( E \) is the modulus of elasticity and \( A \) denotes the metallic cross-sectional area of the rope, and vertical (longitudinal) elastic deflections (vibrations) \( x_{car}, x_{cwt} \) of the car and counterweight, respectively, induced by small vertical motions (oscillations) \( s(t) \) of the machine/ traction sheave assembly (see Fig. 3). This can be represented as a base motion excitation and an adverse situation arises when the car/counterweight are excited at their natural frequency and vibrate periodically at large amplitudes. Such adverse resonance condition may occur due to seismic excitations [5], for example.

For the scenario when the base excitation has been introduced when the car with 125% of the rated load is stationary at the bottom landing (see the system calculation above), a simplified model to represent the dynamic behaviour of the system can then be given by equation (11)

\[
M_{eq_{car\,cwt}} \ddot{x}_{car\,cwt} + c_{car\,cwt} \dot{x}_{car\,cwt} + k_{car\,cwt} x_{car\,cwt} = k_{car\,cwt} x_{cwt} + c_{car\,cwt} \dot{x}_{cwt}
\]

where \( M_{eq_{car\,cwt}} \) represent the well equivalent mass at the car/counterweight sides. The quantities \( k_{car\,cwt} = n_m EA / L_{car\,cwt} \) denote the coefficients of elasticity, where \( L_{car\,cwt} \) define the length of the ropes, at the car/counterweight sides, respectively. Viscous friction model is used to quantify the amount of friction in the well, and \( c_{car\,cwt} \) represent the coefficients of viscous friction at the car/counterweight sides, respectively.

Considering that the base excitation is harmonic \( s = s_{\max} \sin \Omega t \), equations (11) can be re-written as

\[
\ddot{x}_{car\,cwt} + 2 \zeta_{car\,cwt} \omega_{car\,cwt} \dot{x}_{car\,cwt} + \omega_{car\,cwt}^2 x_{car\,cwt} = \frac{s_{\max}}{\zeta_{car\,cwt}} \left( \omega_{car\,cwt}^2 \sin \Omega t + 2 \zeta_{car\,cwt} \omega_{car\,cwt} \Omega \cos \Omega t \right)
\]
where \( \omega_{\text{car}/\text{cwt}} \) denote the natural frequencies of vibrating masses at the car side/counterweight side and \( \zeta \) is the damping ratio [6]. Equations (12) can be solved for the dynamic responses (vibrations) \( x_{\text{car}/\text{cwt}}(t) \).

\[
T_{\text{car}/\text{cwt}} = T_{0\text{car}/\text{cwt}} + k_{\text{car}/\text{cwt}} x_{\text{car}/\text{cwt}}
\]

where \( T_{0\text{car}/\text{cwt}} \) are the static/quasi-static tensions in the ropes.

The vibration effects on the dynamic traction ratio are then evaluated by considering the dynamic tensions in the ropes as

\[
T_{\text{car}/\text{cwt}} = T_{0\text{car}/\text{cwt}} + k_{\text{car}/\text{cwt}} x_{\text{car}/\text{cwt}}
\]

The rope lengths are then determined as \( L_{\text{car}} = H + l_{\text{head}} \), \( L_{\text{cwt}} = l_{\text{head}} \), and the static tensions \( T_{0\text{car}/\text{cwt}} \) are given by equations (9). By considering that the modulus of elasticity of a stranded wire rope with a fibre core lies in the range of \((0.7 \sim 1.0) \times 10^5 \text{ N/mm}^2\) the longitudinal elasticity of one rope is determined as \( EA = 5040.5 \text{ kN} \) where \( E = 0.85 \times 10^5 \text{ N/mm}^2 \) and \( A = 59.3 \text{ mm}^2 \) (for 13 mm 8×19 S-FC rope [4]) are used. The damping ratio is assumed to be \( \zeta = 0.1 \) so that equations (12) can be solved to determine the dynamic responses, followed by calculation of the dynamic tensions from (13).

The dynamic response is determined from equations (12) by numerical integration. Fig. 4 then shows the dynamic tensions in the suspension ropes when the amplitude of base excitation is \( s_{\text{max}} = 0.15 \text{ mm} \) and the frequency of base excitation is 10 Hz. It should be noted that this frequency is close to the natural frequency of the suspension ropes at the counterweight side, which results in vibrations that may compromise traction leading to counterweight jumps [1]. A plot of the corresponding dynamic traction ratios is shown in Fig. 5.

**Figure 3: Simplified model of a lift system subjected to base excitation**

The vibration effects on the dynamic traction ratio are then evaluated by considering the dynamic tensions in the ropes as
It is evident from Fig. 5 that after about 0.29 s the dynamic traction ratios reach the critical value and the system might instantaneously be subjected to traction problems, despite the fact that the standard system analysis predicted that traction is maintained.

4 CONCLUDING REMARKS

The system analysis involves calculations that follow the safety code requirements. These calculations are essential to design a system which complies with accepted safety standards. With a number of commercial/custom-designed software tools/programs to automate the calculations available, the designer is able to arrive at desired results for standard scenarios. However, correct understanding of engineering principles and the assumptions applied in the safety standard formulae is essential to understand the limitations of the results. This aspect is demonstrated through the analysis of resonance condition scenario when the dynamic traction, to comply with European safety standards, needs to be evaluated by rigorous engineering procedure.
The systems analysis and design of lifts (elevators): the models and assumptions appraised

REFERENCES


BIOGRAPHICAL DETAILS

Stefan Kaczmarczyk has a Master’s degree in Mechanical Engineering and he obtained his doctorate in Engineering Dynamics. He is Professor of Applied Mechanics and Postgraduate Programme Leader for Lift Engineering at the University of Northampton. His expertise is in the area of applied dynamics and vibration with particular applications to vertical transportation and material handling systems. Professor Kaczmarczyk has published over 100 journal and international conference papers in this field. He is a Chartered Engineer, a Fellow of the Institution of Mechanical Engineers and a Fellow of the Higher Education Academy.

Phil Andrew has a Master’s Degree in Control Systems Engineering from the University of Warwick. He joined the Express Lift Co. Ltd in 1978 where, over the next 18 years he held a range of senior engineering positions with the company. In 1996 he joined the lift engineering group at the University of Northampton. He led the team who developed the Northampton MSc in Lift Engineering, and then the Foundation Degree in Lift Engineering. In 2003 he took over as Divisional Leader for Engineering in the School of Technology and Design. From the year 2000 until his retirement in 2004, he served on the National Interest Review Committee for the ASME/ANSI A17 Code Committee and represented the University on Committee MHE/4 of the British Standards Institution.