The Study of Hoisting System for Vertical Shaft Construction Without the Protection of Guided-cable

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Abstract. The sinking bucket, known as the hoisting conveyance of the system, is used to transport the waste pile, water, miners and sinking equipment. In the case of a construction shaft when the sinking bucket is lowered downward from the sinking platform to the bottom of a well and it needs to pass through the sinking platform, the bucket is segregated from a guided carriage and it descends or rises without the protection of the guided-cable. This adverse condition is a common phenomenon in the mines vertical shaft. The hoisting rope is a steel wire rope with a low damping, so it is easy to cause oscillating resonance. This study is concerned with the theoretical modeling and simulation verification of the oscillating resonance of the sinking bucket and hoisting cable. A large transient amplitude and steady-state oscillation of the payload may often occur which may result in emergency scenarios. It will threaten the safety of the miners' lives and delay the process of construction of vertical shaft. Thus, the research on the dynamic characteristics of the sinking bucket without the protection of guided-cable is necessary.

1 INTRODUCTION

The flexible steel wire rope is one of the most popular flexible mechanical structures used in engineering due to its advantages in lightweight, lower cost and higher sensitivity, that has been extensively studied both theoretically and practically in recent decades[1, 2]. The translating media with variable length is widely applied in modern industrial fields, such as high elevator cables[3, 4], cable-driven parallel sinking platform[5-7], tethered satellite[8, 9], gantry crane[10, 11] and mining hoists[12, 13]. Bao[14] studied longitudinal vibration of flexible hoisting systems with time-varying length, the governing equations are developed employing the extended Hamilton’s principle considering mutual influence of the rigid motion and deformation of flexible hoisting systems. Kumaniecka[15] investigated analysis of parametric resonance of the longitudinal-transversal vibrations of a rope in a non-linear set of partial differential equations with varying length. Hoisting system for vertical shaft construction includes hoisting cable, guided-cable, sinking platform, and bucket. Due to the fact that the lateral stiffness of the guided rope is smaller, the lateral response of the moving hoisting bucket in cable-guided hoisting system for construction shafts is investigated by Wang [16].

However, the bucket segregated from a guided carriage when it is through the sinking platform and descends or rises without the protection of guided-cable (Fig.1). The model in Fig. 1(a) shows the ‘bucket’ as a concentrated mass m1. Thus, this model represents a simple pendulum, with the mass m_1 suspended on the cable of length $l(t)$. On the other hand, Fig. 1(b) shows the ‘bucket’ as a double pendulum with the concentrated mass m_2 representing the payload. The bucket double pendulum together with the payload form a double pendulum with time-varying length and their oscillations induced by external disturbances that is a great security risk. The flexible nature of the physical structure causes oscillating resonance and degrades their effectiveness and safety. This study is concerned with the theoretical modeling the oscillating resonance of the bucket and hoisting cable in the plane. Hoisting systems without the protection of a guided-cable are highly complex dynamic systems that require detailed simulation and analysis in order to achieve acceptable levels of swing angle.

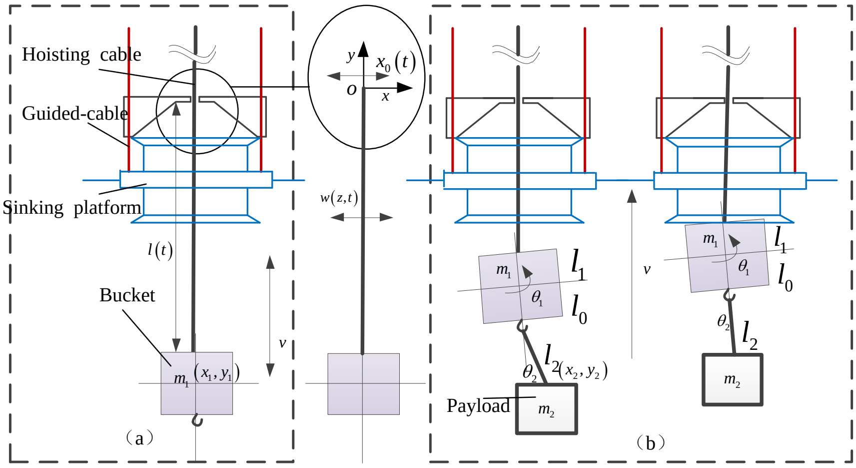

Figure 1 The schematic diagram of hoisting system without guided-cable: (a) Single pendulum; (b) Double pendulum;

2 MODELING

Frame Oxy is the fixed inertia frame (Fig.1), $w(y,t)$ is the position of the flexible cable with respect to the frame Oxy at the position y for time t. The lateral displacement of the cable end is denoted as x. l_1 and l_0 denote the length of the center of the bucket to the upper and lower contact point. l_2 denotes the length of the lower contact point of the bucket to the upper contact point of the payload. θ_1 and θ_2 are the swing angle of the bucket and the payload with respect to the vertical axis, pendulum length, respectively(Fig.1(b)) , respectively. I_1 and I_2 are the inertia of the bucket and the payload, respectively. m_1 and m_2 are the mass of the bucket and the payload, respectively; ρ is the cable density; g is the gravitational acceleration, T_s is cable tension; d_i is constant damping coefficient. Their displacements are expressed as

$$
\begin{aligned}
&x_1 = x + l_1 \sin\theta_1, y_1 = -l - l_1 \cos\theta_1 \\
&x_2 = x + (l_1 + l_0)\sin\theta_1 + l_2 \sin\theta_2, y_2 = -l - (l_1 + l_0)\cos\theta_1 - l_2 \cos\theta_2
\end{aligned}
\tag{1}
$$

Its corresponding velocity can be obtained by taking the derivative of position with respect to time as following:

$$
\begin{aligned}
&\dot{x}_1 = \dot{x} + l_1\dot{\theta}_1 \cos\theta_1, \dot{y}_1 = -\dot{l} + l_1\dot{\theta}_1 \sin\theta_1 \\
&\dot{x}_2 = \dot{x} + (l_0 + l_1)\dot{\theta}_1 \cos\theta_1 + l_2\dot{\theta}_2 \cos\theta_2, \dot{y}_2 = -\dot{l} + (l_0 + l_1)\dot{\theta}_1 \sin\theta_1 + l_2\dot{\theta}_2 \sin\theta_2
\end{aligned}
\tag{2}
$$

The kinetic energy, the potential energy and dissipative energy of the system can be expressed as

$$
T = \frac{1}{2}m_1\left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{1}{2}m_2\left(\dot{x}_2^2 + \dot{y}_2^2\right) + \frac{1}{2}\rho\int_0^l \left(\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y}\right)^2 dy + \frac{1}{2}\sum_{i=1}^{2} I_i\dot{\theta}_i^2 \tag{3}
$$

$$
V = m_1 g\left(l + l_1\left(1 - \cos\theta_1\right)\right) + m_2 g\left(l + (l_1 + l_0)(1 - \cos\theta_1) + l_2\left(1 - \cos\theta_2\right)\right) + \rho g\int_0^l y dy + \int_0^l T_s \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 dy \tag{4}
$$

$$
D = \frac{1}{2}d_0\dot{x}^2 + \frac{1}{2}d_1\dot{\theta}_1^2 + \frac{1}{2}d_2\dot{\theta}_2^2 + \frac{1}{2}d_l\int_0^{l(t)}\left(\frac{\partial w}{\partial t}\right)^2 dy \tag{5}
$$

The solutions of lateral displacement are assumed in the form:

$$
w(y,t) = \sum_{j=1}^{n}\psi_j q_j^{\ c} + \left(1 - \frac{y}{l}\right)x_0(t) \tag{6}
$$

where q_j^c is the generalized coordinate, $\psi_j(\xi)=\sqrt{2}\sin\left(\left(j-1/2\right)\pi\xi\right)$ is the mode functions, $x_0(t)$ is the boundary excitation at the guided carriage (Fig.1) that can be assumed to be a simple harmonic function.

The government equation can be obtained from the action quantity into the Lagrange equation [17, 18]

$$\frac{\mathrm{d}}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}+\frac{\partial D}{\partial \dot{q}_j}+\frac{\partial V}{\partial q_j}=\sum_{i=1}^{N}\lambda_i\frac{\partial g_i}{\partial q_j},\left(j=1,2\cdots n\right) \tag{7}$$

In which, $\boldsymbol{q}=\left[\boldsymbol{q}^c,x,\theta_1,\theta_2\right]^{\mathrm{T}}$; $\boldsymbol{g}\in R^{N\times 1}$ is the vector of geometric boundary conditions hoisting cable and the bucket, and λ_i is the Lagrange multipliers. From Fig. 1, only one constraint condition is given by $g_1=x-w(l,t)=0, N=1$.

After ignoring high order terms, the dynamic equations (7) can be rewritten as

$$\begin{bmatrix}\boldsymbol{M}_c & \\ & \boldsymbol{M}_p\end{bmatrix}\ddot{\boldsymbol{q}}+\begin{bmatrix}\boldsymbol{C}_c & \\ & \boldsymbol{C}_p\end{bmatrix}\dot{\boldsymbol{q}}+\begin{bmatrix}\boldsymbol{K}_c & \\ & \boldsymbol{K}_p\end{bmatrix}\boldsymbol{q}=\begin{bmatrix}\boldsymbol{F}_c\\ \boldsymbol{0}_{3\times 1}\end{bmatrix}+\boldsymbol{\lambda}^{\mathrm{T}}\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{q}} \tag{8}$$

where $\boldsymbol{M}_{\square}$ denotes the inertia matrix, $\boldsymbol{C}_{\square}$ is the damping matrix, $\boldsymbol{K}_{\square}$ denotes the stiffness matrix, and $\boldsymbol{F}_{\square}$ represents the force input vector. These matrixes are expressed as follows:

$$\boldsymbol{M}_c=\rho l\boldsymbol{M}_1,\quad \boldsymbol{C}_c=\rho v\left(\boldsymbol{C}_1+\boldsymbol{C}_2-\boldsymbol{C}_2^{\mathrm{T}}\right)+d_l l\boldsymbol{C}_1,\quad \boldsymbol{K}_c=\rho a\boldsymbol{K}_1-\rho v^2/l\boldsymbol{K}_2+\left(\left(g-a\right)\sum_{k=1}^{2}m_k/l\boldsymbol{K}_3+\left(g-a\right)\rho\boldsymbol{K}_4\right)-d_l v\boldsymbol{K}_5$$

$$\boldsymbol{F}_c=-\rho\left(v^2x_0(t)/l-v\dot{x}_0(t)\right)\boldsymbol{F}_1-\rho\left(l\ddot{x}_0(t)-ax_0(t)\right)\boldsymbol{F}_2-\left(g-a\right)\left(\sum_{k=1}^{2}m_k/l\boldsymbol{F}_3+\rho\boldsymbol{F}_4\right)x_0(t)$$

$$\boldsymbol{M}_p=\begin{pmatrix} m_1+m_2 & \cos\theta_1\left(l_1m_1+\left(l_0+l_1\right)m_2\right) & l_2m_2\cos\theta_2\\ \cos\theta_1\left(l_1m_1+\left(l_0+l_1\right)m_2\right) & I_1+l_1^2m_1+\left(l_1+l_0\right)^2m_2 & l_2m_2\cos\left(\theta_1-\theta_2\right)\left(l_0+l_1\right)\\ l_2m_2\cos\theta_2 & l_2m_2\cos\left(\theta_1-\theta_2\right)\left(l_0+l_1\right) & m_2l_2^2+I_2\end{pmatrix}$$

$$\boldsymbol{C}_p=\begin{pmatrix} d_0 & 0 & 0\\ 0 & d_1 & 0\\ 0 & 0 & d_2\end{pmatrix},\boldsymbol{K}_p=\begin{pmatrix} 0 & 0 & 0\\ 0 & gl_1m_1+gm_2\left(l_0+l_1\right) & 0\\ 0 & 0 & gl_2m_2\end{pmatrix},$$

where, $\boldsymbol{M}_i,\boldsymbol{C}_i,\boldsymbol{K}_i$ and $\boldsymbol{F}_i$ are corresponding the mass, damping , stiffness coefficient matrices and the coefficient force vectors, respectively.

3 SIMULATION ANALYSIS

Suppose the hoisting system is an ideal power source and the bucket moving with the expected motion trajectory. In most situations, the bucket is not only loaded inside and usually needs, in practice, to hoist heavy equipment on the bottom of the bucket. When the bucket is raised to a certain height, the payload is hung at the bottom of the bucket with a certain inclination angle. In addition, for safety reasons, the initial displacement of the hoisting system should be limited in a certain range. Table 1 shows the main parameters of the system and the initial conditions are the angles $\theta_1(0),\theta_2(0)$.

Table 1 Main parameters and initial conditions

Parameters	Physical significance	Value
m_1	The bucket mass	500kg
m_2	The payload mass	500kg
ρ	Cable density	2kg/m
$l(0)$	Initial hoisting cable length	30m
l_0, l_1, l_2	Geometry length	0.5m,0.5m,1.5m
I_1, I_2	Inertia moment	200kgm^2, 200kgm^2
$\theta_1(0), \theta_2(0)$	Initial swing angle	0° ,10°
d_i, d_l	Damping coefficient	1.12,0.005

The difference between a double pendulum and a single pendulum as showed in Fig.2. Compared with the double pendulum, lateral amplitude of the bucket is quite small. When the bucket is separated, it is usually just below the sinking platform and subjected to the excitation of a guided carriage and external disturbances. The swing motion of the hoisting cable and the bucket is a substantial low-frequency oscillation. They swing more severely and the residual swing is sometimes difficult to dissipate, which may knock and damage the other parts of sinking platform. Neglecting the high order term model can still describe the real behavior of a double pendulum hoisting of the small angle assumption. However, it can't fully reflect the overall influence of the lateral vibration of the swing of hoisting cable.

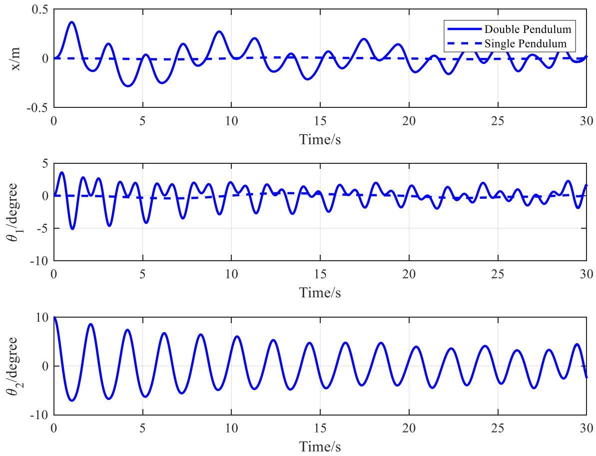

Figure 2 The difference between a double pendulum and a single pendulum: Dotted line: a single pendulum; Soild line: a double pendulum.

The two natural frequencies of the double pendulum system can be obtained by the empirical formula [19] [20]. The natural frequencies $\omega_{1,2}$ are described as follows

$$\omega_{1,2} = \sqrt{\frac{g}{2}\left((1+R)\left(\frac{1}{l}+\frac{1}{l_2}\right) \mp \sqrt{(1+R)^2\left(\frac{1}{l}+\frac{1}{l_2}\right)^2 - 4\frac{1+R}{l \cdot l_2}}\right)}, \tag{9}$$

where, $R = m_1 / m_2$ is the mass ratio of payload and bucket. It can be seen that the two nature frequencies not only depend on the length of the cables but also depend on the mass ratio.

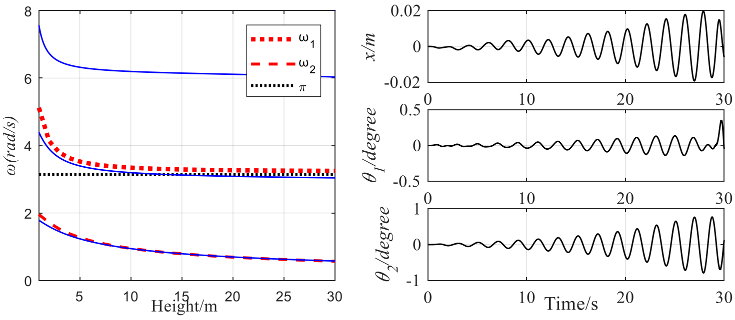

Figure 3 The frequency of swing angle and hoisting cable

The frequency of swing angle and lateral frequency hoisting cable as showed in Fig.3. One can obtain a natural frequency ω from

$$\left|\omega^2\begin{bmatrix}\boldsymbol{M}_c & \\ & \boldsymbol{M}_p\end{bmatrix}-\begin{bmatrix}\boldsymbol{K}_c & \\ & \boldsymbol{K}_p\end{bmatrix}\right|=0 \tag{10}$$

The first two order frequencies of the nonzero eigenvalues of the matrix are in a agreement with the frequencies of the swing angle. The second order frequency of swing angle is slightly higher than the result of matrix nonzero eigenvalues that's because the Eq.(9) does not take into consideration the mass of the hoisting cable. When the excitation frequency is πrad/s , so it is easy to cause oscillating resonance in Fig.3. The third order frequency is the lateral frequency of the hoisting cable higher than the swing angle frequency.

A set of initial values of the swing angle is set, and four-order Runge-Kutta numerical computing method is used to solve the equations governing the response of the system. Its plane trajectory can be obtained. The numerical results demonstrate that the plane trajectory of a double pendulum with different initial angles, the displacement boundary shown in the black dotted line. For example, in Fig. 4, the largest amplitude of payload reaches over 0.32m ($\theta_2(0)=15^\circ$) beyond the boundary line. Its displacement is not greater than the boundary line within the dis-placement boundary never exceeds with 10°initial angles. The position of the payload must be kept in the circle of the central point during the hoisting.

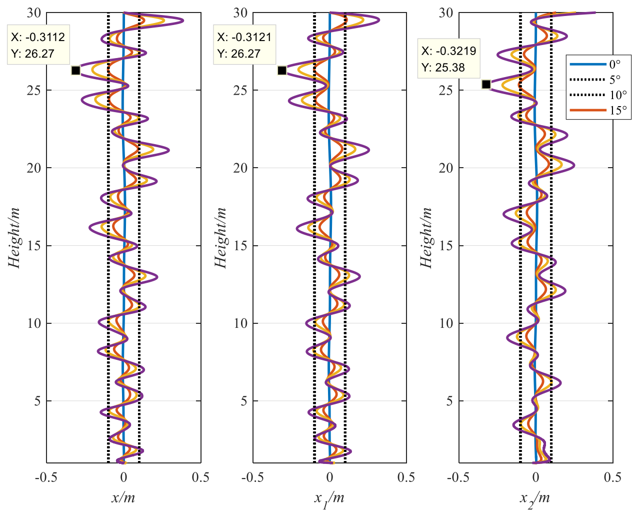

Figure 4 Comparison of the response of the system with different initial angles

4 CONCLUSION

This paper presented theoretical analyses of the hoisting system dynamics for vertical shaft construction without the protection of a guided-cable. The equations of motion of the hoisting system for vertical shaft construction comprising a bucket, a pay-load and hoisting cable excited by external disturbance are derived in this paper. The natural frequencies of the systems depend on the length of the cable, and the bucket-payload mass ratio and is used to predict the response of the system. Frequency characteristics analyses show that both the hoisting cable mass has effects on the second-mode frequency. The external disturbance of the payload will cause oscillation in a substantial low-frequency. Numerical simulation results show that the energy is passed from the payload to the bucket and hoisting cable. Once the initial inclination angle continues to grow, collision phenomenon between the bucket and the other parts of sinking platform in the hoisting process might occur which may lead to the swing of the payload and damage to the components of sinking platform.

5 ACKNOWLEDGMENTS

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BIOGRAPHICAL DETAILS

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