Dynamic Behavior of Traction System with Tension at the Pulley of Compensating Rope

Wang Lei and Cao Guohua

School of Mechatronic Engineering, China University of Mining and Technology, Xuzhou, China

Keywords: Traction system, dynamics, tension pulley

Abstract: With high capacity and low cost, the traction system is used in various lifting applications. Caused by the effect of hoisting and compensation rope with time-varying length, the longitudinal vibration of the system is varying moderately during the period of operation. In this paper, the dynamic model of the traction system with compensating rope is established based on Lagrange equations of the first kind. The compensating rope is tensioned by a pulley with external force working on it. The dynamics resonance characteristics under different working conditions of bidirectional conveyances and tensioning pulley can be obtained, which can provide reference for the design of long- travel and high-speed hoisting systems.

1 INTRODUCTION

Due to their ability to resist relatively large axial loads, ropes have been widely used in many different applications to support structures, conduct signals, and carry payloads. Many researchers have concentrated on the longitudinal vibrations of the hoisting cable or container for decades. Kaczmarczyk and Ostachowicz [1,2] investigated the longitudinal responses of a hoisting cable in the mine hoisting system and a compensation cable in a high-speed elevator. Ren and Zhu [3] presented the longitudinal and lateral vibrations of a moving two-cable one-rigid-body-car system, in which the rotation of the car is considered. For the spatial discretization of a cable, assumed modes method (AMM) [3] and finite element method [4, 5] are commonly adopted. For the equation of motion, Lagrange's equations [6, 7] or Hamilton's principle can be selected. Terumichi Y. [8] researched the nonstationary vibration of a string with a constant hoisting speed, and the analytical results showed that the axial velocity of the string influenced the peak amplitude of the string vibration at the passage through resonances. But in above references, only single side dynamic model is established and the compensating rope is ignored, so the effect of tensioning pulley and tensioning force on the traction system can't be investigated.

2 MODEL DESCRIPTION

As shown in Fig. 1, the traction system with tension at the pulley is investigated, the length of hoisting cable is donated as $l_i\left(t\right)$, and their corresponding velocity and acceleration can be expressed as $v_i\left(t\right)$ and $a_i\left(t\right)$, respectively.

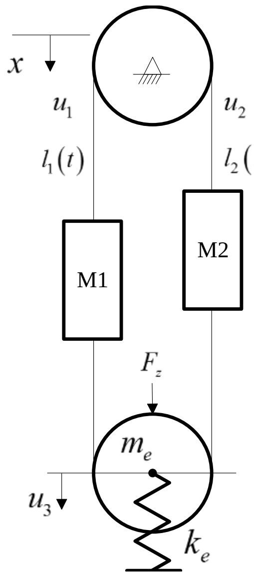


Figure 1 Traction system with tension at the pulley

3 MATHEMATICAL MODEL

The model was established based on the energy method, and the kinetic energy is given as followed:

$$K_e = \frac{1}{2}\int_0^{LL} \rho_1 \left(\frac{Du_1}{Dt} + v_1\right)^2 dx + \frac{1}{2}\int_0^{LL} \rho_2 \left(\frac{Du_2}{Dt} + v_2\right)^2 dx + \frac{1}{2} m_e \dot{u}_3^2 \tag{1}$$

The potential energy of the traction system is a function of the vibration displacement, which is given as followed:

$$E_e = \int_0^{LL} \left[T_1(x,t)\varepsilon_1 + \frac{1}{2} EA\varepsilon_1^2 \right] dx + \int_0^{LL} \left[T_2(x,t)\varepsilon_2 + \frac{1}{2} EA\varepsilon_2^2 \right] dx + \frac{1}{2} k_e u_3^2 \tag{2}$$

$$E_g = -\int_0^{LL} \rho_1 g \cdot u_1(x,t) dx - \int_0^{LL} \rho_2 g \cdot u_2(x,t) dx - m_e g u_3 \tag{3}$$

Where $\rho_i\left(i = 1, 2\right)$ is the linear density of the lifting cable with concentrated inertia elements m_i attached to it. m_i is the hoisting conveyance mass. $T_i(x,t)$ is the tension at position *x* of the cable. $u_i\left(x,t\right)$ is the longitudinal vibration at position x of the cables; u_3 is the longitudinal vibration of the tensioning pulley, m_e is the quality of the tensioning pulley, k_e is the stiffness of the tensioning spring, LL is the distance between traction pulley and tensioning pulley.

ε is the elastic strain of the cable. Caused by the neglection of transverse vibration, the expression of ε can be obtained as follows:

$$\varepsilon = \frac{1}{2} u_x \tag{4}$$

After the normalization, the kinetic energy, the elastic potential energy and the gravitational potential energy of the system are brought into the Lagrange equation of the first kind.

$$\frac{d}{dt}\frac{\partial K_e}{\partial \dot{q}_i} - \frac{\partial K_e}{\partial q_i} + \frac{\partial\left(E_e + E_g\right)}{\partial q_i} = Q_i + \sum_{k=1}^{n} \lambda_k \frac{\partial g_k}{\partial q_i} \tag{5}$$

In this paper, the constraint conditions at the tensioning pulley can be defined as:

$$g_1 : u_1\left(LL,t\right)+u_2\left(LL,t\right)=2u_3 \tag{6}$$

Using AMM, the solution can be expressed as:

$$u(\xi,t)=\sum_{i=1}^{n}U_i\left(\xi\right)q_i\left(t\right) \tag{7}$$

Where U_i is the assumed mode and set as $\sin(\frac{2i-1}{2}\xi)$,$\left(0\le\xi\le1\right)$. q_i is the generalized displacement and set as $\left[q_1,q_2,u_3\right]^{\mathrm{T}}$. As shown in figure 1, q_1 is the generalized displacement of the left rope, q_2 is the generalized displacement of the right rope, and u_3 is the generalized displacement of the tensioning pulley.

Substituting Eqs. (7) into Eq. (5), The equation of motion can be reduced as followed:

$$\begin{aligned} &M\ddot{Q}+C\dot{Q}+KQ=F+G^{\mathrm{T}}\lambda \\ &g(q,t)=0 \end{aligned} \tag{8}$$

Where G is the Jacobian matrix of the constraint equations, and $\lambda=\left(\lambda_1,\lambda_2,...,\lambda_c\right)$ are Lagrangian multipliers, which denote the constraint forces between two hoisting cables and the tensioning pulley, $Q=\left[q_1,q_2,u_3\right]^{\mathrm{T}}$ is a vector of generalized displacement. By derivation, the forcing term F can be obtained as follows.

$$\boldsymbol{F}_{u,i}=\rho_i v_i^2\int_0^1 U_j'\left(\xi\right)\mathrm{d}\xi-\rho_i LLa_i\int_0^1 U_j(\xi)\mathrm{d}\xi-T_i\left(\xi,t\right)\int_0^1 U_j'\left(\xi\right)d\xi \ ,i=1,2 \tag{9}$$

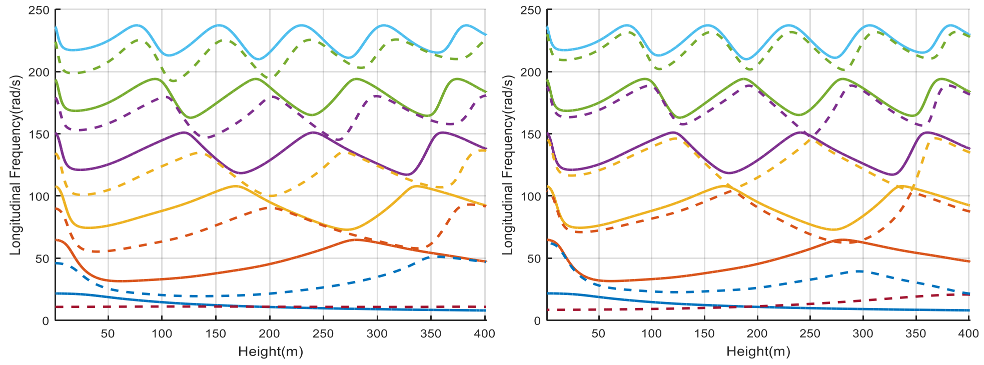
4 NUMERICAL CALCULATION

The parameters used in the numerical simulation are as listed in Table.1.

Table 1 Fundamental parameters of the system

Parameter	Definition	Value
$\boldsymbol{H}$	Travel Height	401 m
$\boldsymbol{V}$	Running Velocity	10 m/s
$\boldsymbol{A}$	Running Acceleration	$2\ \mathrm{m/s^2}$
$\boldsymbol{\rho}$	Rope linear mass density	22.6 kg/m
$\boldsymbol{E_s}$	Rope Modulus of elasticity	1.2×10^{11} Pa
$\boldsymbol{A_s}$	Rope Cross section area	$6.94\times10^{-3}\ \mathrm{m^2}$
$\boldsymbol{M_z}$	Conveyance mass	1.5×10^{4} kg

The frequency characteristics of the two-conveyance system can be obtained by numerical calculation, which is shown in Fig 2. The kinetic equation can be obtained using AMM and the mass matrix M and the stiffness matrix K can be extracted. Finding out eigenvalues of the matrix $\sqrt{K/M}$, and the natural frequencies of each order can be obtained by sorting the eigenvalues at corresponding time.



(a) (b)

Figure 2 The frequency characteristics of the bilateral hoisting system with tensioning pulley and without tensioning pulley

Figure 2-(a) stands for the descend progress of M_1, and Figure 2-(b) is in the opposite direction. In the figures above, the solid lines are the natural frequency of lifting system without tensioning pulley, and the dotted lines are the natural frequency of lifting system with tensioning pulley. From the result above, we can see the natural frequencies of each order of lifting system are decreasing correspondingly caused by the tensioning pulley.

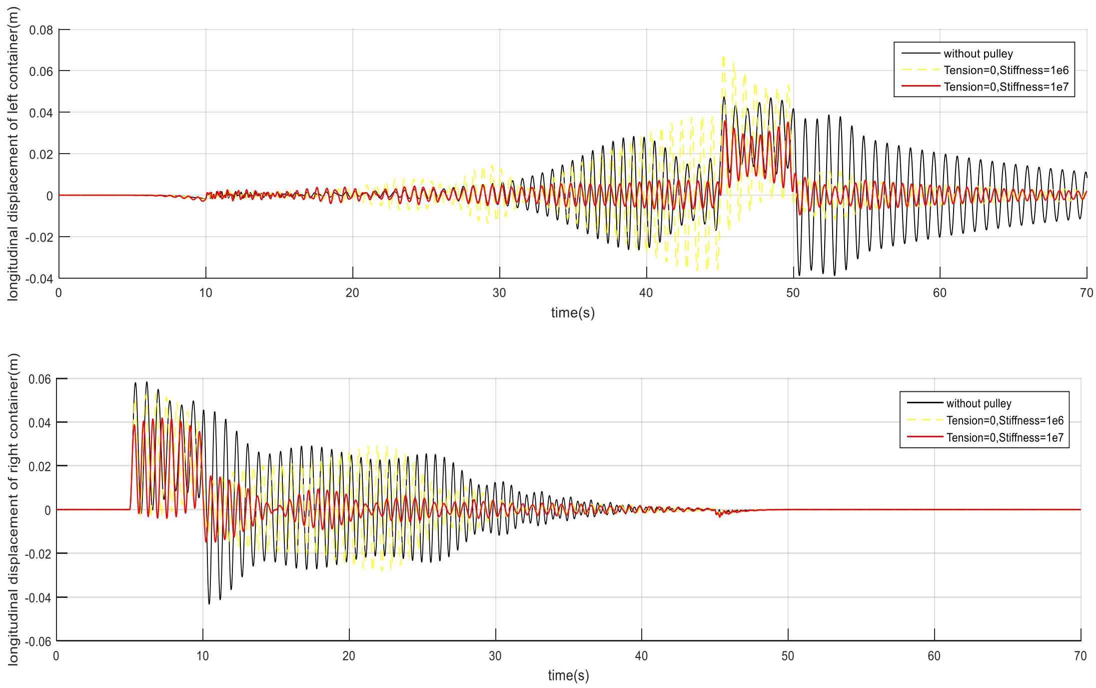


Figure 3 The longitudinal displacement of bilateral conveyance with different tensioning stiffness

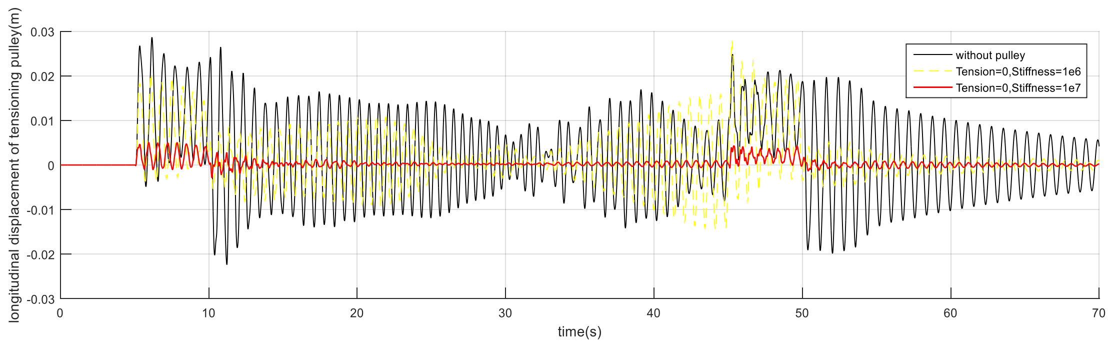


Figure 4 The longitudinal displacement of tensioning pulley with different tensioning stiffness

Firstly, the impact of the tensioning spring is discussed by comparing the numerical solution results of different stiffness working on the pulley. As shown in Fig 3 and 4, the curve trend of hoisting conveyance with and without tensioning pulley are basically similar. By comparing the red and black line in above pictures, the vibration amplitude of hoisting conveyance without tensioning stiffness is much higher than that with a tensioning spring of proper stiffness working on the pulley. However, by comparing the yellow and black line in above pictures, it can be seen that the tension spring can't get its effect if the tensioning stiffness isn't selected in the appropriate range. The results show that the tensioning stiffness can achieve the vibration amplitude's suppression of the hoisting conveyance. Similarly, as shown in Fig 4, the longitudinal displacement of the tensioning pulley is also decreased by proper tensioning stiffness working on it.

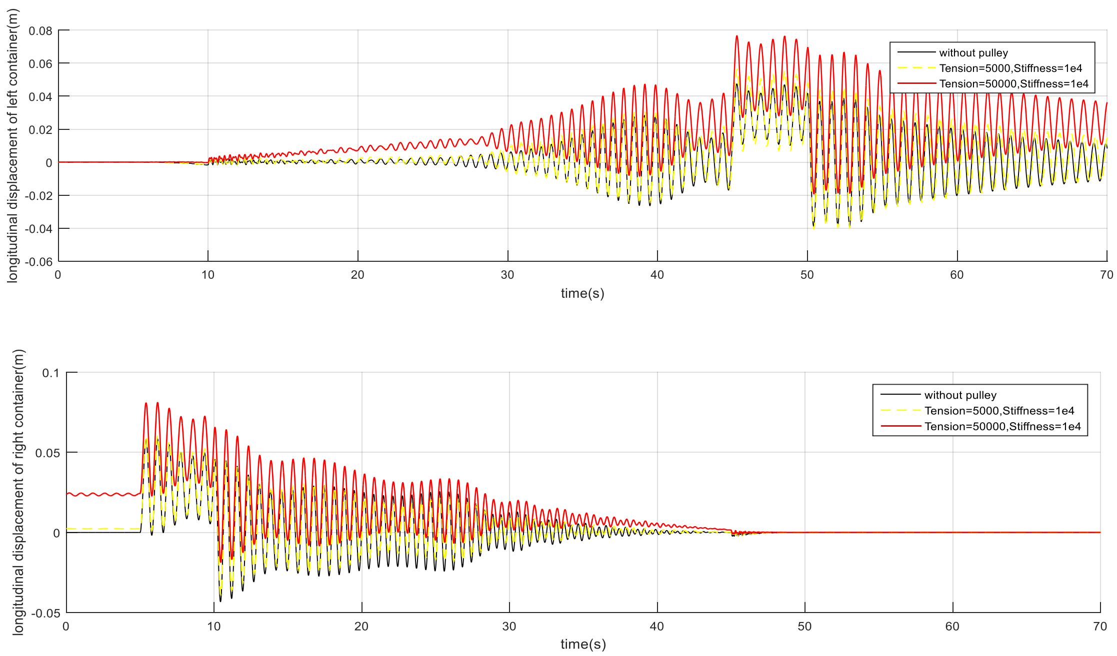


Figure 5 The longitudinal displacement of bilateral conveyance with different tension forces

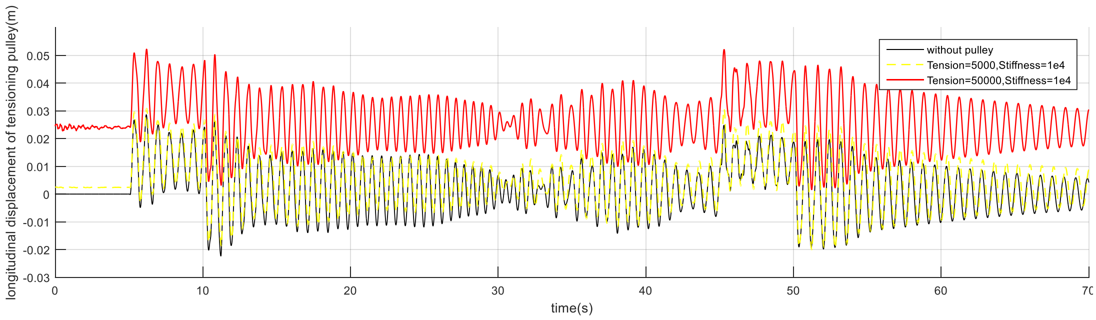


Figure 6 The longitudinal displacement of tensioning pulley with different tension forces

Lastly, the impact of tensioning force is also discussed by comparing different tensioning forces working on the pulley. As shown above, the dynamic amplitude of the conveyances and tensioning pulley aren't substantial changed with the increase of tensioning forces. The difference among different tensioning forces is the change of cables' static displacement at the conveyances and tensioning pulley caused by the tension forces. It can be concluded that the tensioning forces working on the pulley will not affect the longitudinal vibration shape of the traction system.

CONCLUSION

In this paper, the dynamics behavior of the traction system with tension at the pulley is investigated. The cables are spatially discretized using the AMM and the equations of motion are established by Lagrange equations of the first kind. From the numerical results, it can be concluded that the tensioning spring with proper stiffness can suppress the longitudinal vibration at the conveyance and tensioning pulley. However, the tensioning force working on the pulley at the bottom of the traction system can't play an effective role in the vibration suppression. Above all, this theoretical model can predict the response of the traction system, which will lay the foundation for the longitudinal vibration control of elevator hoisting system.

ACKNOWLEDGMENTS

This work was supported by the National Key Research and Development Program

(2016YFC0600901) and National Natural Science Foundation of China (51475456).

REFERENCES

[1] Kaczmarczyk S., Ostachowicz W. Transient vibration phenomena in deep mine hoisting cables. Part 1: Mathematical model. Journal of Sound and Vibration, Vol. 262, Issue 2, 2003, p. 219-244.

[2] Kaczmarczyk S., Picton P. The prediction of nonlinear responses and active stiffness control of moving slender continua subjected to dynamic loadings in a vertical host structure. International Journal of Acoustics and Vibration, Vol. 18, Issue 1, 2013, p. 39-44.

[3] Ren H., Zhu W. D. An accurate spatial discretization and substructure method with application to moving elevator cable-car systems. Part II: Application. Journal of Vibration and Acoustics, Transactions of the ASME, Vol. 135, Issue 5, 2013, p. 51037.

[4] Wang P. H., Fung R. F., Lee M. J. Finite element analysis of a three-dimensional underwater cable with time-dependent length. Journal of Sound and Vibration, Vol. 209, Issue 2, 1998, p. 223-249.

[5] Du J. L., Cui C. Z., Bao H., Qiu Y. Y. Dynamic analysis of cable-driven parallel manipulators using a variable length finite element. Journal of Computational and Nonlinear Dynamics, Vol. 10, Issue 1, 2015, p. 11013.

[6] Dvorak R., Freistetter F. Chaos and Stability in Planetary Systems. Springer-Verlag, Berlin, Germany, 2005.

[7] Zhu W. D., Ren H. An accurate spatial discretization and substructure method with application to moving elevator cable-car systems. Part I: Methodology. Journal of Vibration and Acoustics, Transactions of the ASME, Vol. 135, Issue 5, 2013, p. 051036.

[8] Terumichi Y., Ohtsuka M., Yoshizawa M., Fukawa Y., Tsujioka Y. Nonstationary vibrations of a string with time-varying length and a mass-spring system attached at the lower end. Nonlinear Dynamics, Vol. 12, Issue 1, 1997, p. 39-55.

BIOGRAPHICAL DETAILS

Wang Lei is a Ph.D. candidate from China University of Mining and Technology, Xuzhou, China, in 2018. His current research interests include hoisting system dynamics and vibration control.

Guohua Cao is a professor at China University of Mining and Technology. His major research includes mechanical vibration and lifting dynamics.