Coupled Vibration of Rope-guided Hoisting System Under Multiple Constraint Conditions

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Abstract. In addition to being used as hoisting ropes, wire ropes are also used as guiding rails to provide guidance for hoisting conveyances in many Chinese hoisting systems of hundred meters or even a kilometer mine shaft and outdoor long-travel lifting systems. The vibratory response in this rope-guided hoisting system differs from flexible rails and gradually attracts researchers' attention. Therefore, modeling of coupled vibration of time-varying hoisting systems with four rope guides is presented by energy method in this paper and the system vibration equation is established with Lagrange multiplier method under multiple constraint conditions. Lateral and torsional coupling vibrations are analyzed under hoisting rope eccentricity, tension and density differences between rope guides. These characteristics provide guidelines for the design of double conveyances hoisting systems to prevent conveyances colliding and lay a foundation for future research.

1 INTRODUCTION

Dynamic Characteristics Investigation of hoisting systems is an important task for system design in mine or elevator hoisting systems. Wire ropes used as guiding rails are common substructures in mine hoisting shafts. Different from high-frequency vibration of the system with rigid rails [1], rope-guided system vibrations are characterized by lower-frequency vibrations and greater lateral amplitudes. Wang [2] simplified the guiding rope as a spring-mass module and established a lateral vibration model with two guiding rope constraints in a vertical mine construction system. Based on Wang's model [2], Cao [3, 4] analyzed the system energetic and later regarded guiding rope as continuum and numerically solved the model with Lagrange multiplier method. Wang [5] built a double drum winding hoisting model with two guiding ropes, but the guiding ropes are regarded as a force acted on conveyance. This shows the dynamics of rope-guided systems attract growing attention. Present dynamic models of rope-guided systems are limited to two guiding ropes, while there are four guiding ropes in many Chinese mine shafts. Therefore, dynamic modeling of hoisting systems with four guiding ropes is built based on Lagrange multiplier method in this paper and lateral and torsional coupling responses are revealed. This work is important for safety and stability of working conveyances.

2 MODELING FOR HOISTING SYSTEM

Geometrical model in a rope-guided hoisting system is shown as Fig. 1a). A conveyance, moving at speed v with mass of m, is suspended by a hoisting rope with length of l(t) and constrained by four guiding ropes each with length of L_i and bottom tension of T_{bi} (i=1, 2, 3, 4), as shown in Fig. 1b). Constraints between conveyance and guiding ropes are demonstrated in Fig. 1c), where w_c and u_c and θ are lateral and torsional responses of conveyance in w-u plane with coordinate origin "O", the hoisting point. (A_i, B_i) is coordinate position of the ith guiding rope at constrained point between the guiding rope and conveyance. w_i and u_i are responses of a guiding rope in w and u directions, respectively. Hoisting rope is made by the twisting of several steel wire ropes, thus the torsion from twisting is assumed to be zero and hoisting ropes are regarded as one rope in this model. Torsion of conveyance is only caused by the movement in the directions of w and u. The model is based on the following assumptions: (1) longitudinal vibrations and their coupled vibration with lateral motions are neglected. (2) Conveyance height and revolute motion around lateral axes are neglected.

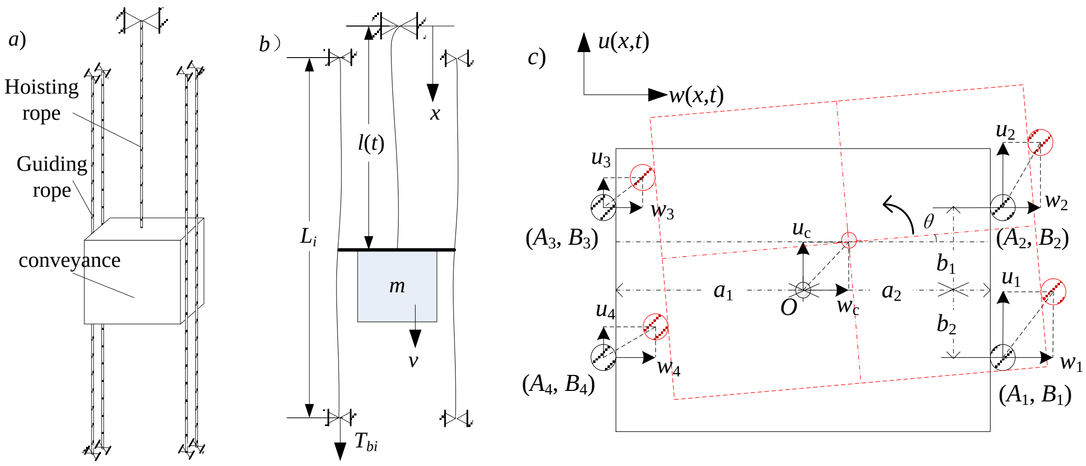

Figure 1 Modeling for hoisting system: *a*) System model, *b*) Mathematical model in vertical direction, *c*) Mathematical model in lateral direction

Kinetic energy T_k and potential energy V_e of the system are:

$$\begin{aligned} T_k &= \frac{1}{2}\int_0^{l(t)} \rho_0 \left[\left(u_{0,t} + v u_{0,x}\right)^2 + \left(w_{0,t} + v w_{0,x}\right)^2 \right] \mathrm{d}x \\ &+ \frac{1}{2}\sum_{i=1}^{4}\int_0^{L_i} \rho_i \left[\left(w_{i,t}\right)^2 + \left(u_{i,t}\right)^2 \right] \mathrm{d}x + \frac{1}{2} m_c \left(\dot{u}_c^{\,2} + \dot{w}_c^{\,2} \right) + \frac{1}{2} J_c \dot{\theta}^2 \end{aligned} \tag{1}$$

$$V_e = \int_0^{l(t)} \left[\rho_0 \left(l - x\right) + m \right] \left(g - a\right) \varepsilon_0 \mathrm{d}x + \sum_{i=1}^{4} \int_0^{L_i} \left[\rho_i g \left(L_i - x\right) + T_{bi} \right] \varepsilon_i \mathrm{d}x \tag{2}$$

where ρ_0 and ρ_i are length density of hoisting rope and the ith guiding rope, respectively. Subscript x and t respectively denote partial derivatives of variables with respect to x and t. Overdot " • " is derivative with respect to t. Lateral strain measure of hoisting rope and the ith guiding rope are given as $\varepsilon_0 = 0.5(w_{0,x}^2 + u_{0,x}^2)$ and $\varepsilon_i = 0.5(w_{i,x}^2 + u_{i,x}^2)$. J_c is rotational inertia for transverse section of conveyance. Since the hoisting rope is subject to an external force at $x(t) = 0$ due to head sheave vibration, lateral excitations in w and u directions are set as $w_0(0, t) = e_1(t)$, $u_0(0, t) = e_2(t)$. Setting vibration of the hoisting rope as two sections of homogeneous and non-homogeneous solutions yields Eq. (3).

$$\begin{aligned} w_0\left(x,t\right) &= \bar{w}_0\left(x,t\right) + \bar{h}_1\left(x,t\right) \\ u_0\left(x,t\right) &= \bar{u}_0\left(x,t\right) + \bar{h}_2\left(x,t\right) \end{aligned} \tag{3}$$

where $\bar{h}_i\left(x,t\right) = e_i\left(t\right)\left(1 - x/l\right)$, ($i$=1,2), The boundary conditions are:

$$\begin{aligned} & w_i\left(0,t\right) = w_i\left(L_i,t\right) = u_i\left(0,t\right) = u_i\left(L_i,t\right) = 0,\ \ i = 1,2,3,4 \\ & \bar{w}_0\left(0,t\right) = \bar{u}_0\left(0,t\right) = 0 \end{aligned} \tag{4}$$

According to Galerkin's method, time-variant domain [0, $l(t)$] of the hoisting rope and [0, L_i] of the guiding rope for x can be converted to fixed domain [0, 1] for ζ and η_i, respectively, where $\zeta = x/l(t)$ and $\eta_i = x/L_i$ (i=1, 2, 3, 4). Any variable () in time-variant domain is expressed with $\tilde{()}$ in fixed domain. The solutions of the system are assumed as following forms:

$$
\begin{aligned}
&\tilde{w}_0(\zeta,t)=\sum_{j=1}^{N_1}\varphi_j(\zeta)\,p_{0,j}(t)\\
&\tilde{u}_0(\zeta,t)=\sum_{j=1}^{N_1}\varphi_j(\zeta)\,q_{0,j}(t)\\
&\tilde{w}_i(\eta,t)=\sum_{j=1}^{N_2}U_{i,j}(\eta)\,p_{i,j}(t),\, i=1,2,3,4\\
&\tilde{u}_i(\eta,t)=\sum_{j=1}^{N_2}U_{i,j}(\eta)\,q_{i,j}(t),\, i=1,2,3,4
\end{aligned}
\tag{5}
$$

where $p_{0,j}$ and $q_{0,j}$ as well as $p_{i,,j}$ and $q_{i,,j}$ are generalized coordinates for the hoisting rope and the *i*th guiding rope respectively in *w* and *u* directions, φ_j and $U_{i,\,j}$ are corresponding vibration mode functions:

$$
\begin{aligned}
&\varphi_j(\zeta)=\sin\left(\frac{2j-1}{2}\pi\zeta\right)\\
&U_{i,j}(\eta_i)=\sin(j\pi\eta_i),\, i=1,2,3,4
\end{aligned}
\tag{6}
$$

According to Fig. 1c), constraint conditions between conveyance hoisting rope and four guiding ropes are:

$$
\begin{aligned}
&g_1=\tilde{w}_c-\tilde{w}_0(l,t)=0\\
&g_2=\tilde{u}_c-\tilde{u}_0(l,t)=0\\
&\begin{cases} g_{2i+1}=w_c-(1-\cos\theta)A_i-B_i\sin\theta-w_i(l,t)\\ g_{2i+2}=u_c+A_i\sin\theta-(1-\cos\theta)B_i-u_i(l,t)\end{cases},\, i=1,2,3,4
\end{aligned}
\tag{7}
$$

where (A_1, B_1)=(a_2, -b_2), (A_2, B_2)=(a_2, b_1), (A_3, B_3)=(-a_1, b_1), (A_4, B_4)=(-a_1, -b_2). Since the conveyance rotation angle is small enough, assumptions sin θ=θ and cos θ=1 are valid to obtain the solution. System vibration are given by Equation (8)-(9).

$$
\frac{d}{dt}\frac{\partial T_k}{\partial \dot{Q}_i}-\frac{\partial T_k}{\partial Q_i}+\frac{\partial V_e}{\partial Q_i}=\sum_{j=1}^{10}\lambda_j\frac{\partial g_j}{\partial Q_i}
\tag{8}
$$

$$
\mathbf{M}\ddot{\mathbf{Q}}+\mathbf{C}\dot{\mathbf{Q}}+\mathbf{K}\mathbf{Q}=\mathbf{F}+\mathbf{G}^{\mathrm{T}}\boldsymbol{\lambda}
\tag{9}
$$

where $\mathbf{Q}$=[$p_{0,1}$,…, $p_{0,N1}$, $q_{0,1}$,…, $q_{0,N1}$, $p_{1,1}$,…, $p_{1,N2}$, $q_{1,1}$,…, $q_{1,N2}$, $p_{2,1}$,…, $p_{2,N2}$, $q_{2,1}$,…, $q_{2,N2}$, $p_{3,1}$,…, $p_{3,N2}$, $q_{3,1}$,…, $q_{3,N2}$, $p_{4,1}$,…, $p_{4,N2}$, $q_{4,1}$,…, $q_{4,N2}$, w_c, u_c, θ]$^{\mathrm{T}}$ is generalized coordinate vector, $\boldsymbol{\lambda}$=[λ_1, λ_2,…, λ_{10}]$^{\mathrm{T}}$ is Lagrange multiplier vector, $G_{j,i}=\partial g_j/\partial Q_i$, **M**, **C**, **K** and **F** are matrices of mass, damping, stiffness and force, respectively. In order to simplify solution, transformation square matrix $\mathbf{T_R}$ is introduced. Thus, $\mathbf{Q}_{\mathrm{T}}=\mathbf{T_R}\cdot\mathbf{Q}$, $\mathbf{M}_{\mathrm{T}}=\mathbf{T_R}\cdot\mathbf{M}\cdot\mathbf{T_R}^{\mathrm{T}}$, $\mathbf{K}_{\mathrm{T}}=\mathbf{T_R}\cdot\mathbf{K}\cdot\mathbf{T_R}^{\mathrm{T}}$, $\mathbf{F}_{\mathrm{T}}=\mathbf{T_R}\cdot\mathbf{F}$, $\mathbf{G}_{\mathrm{T}}=\mathbf{G}\cdot\mathbf{T_R}^{\mathrm{T}}$. Setting $\mathbf{Q}_{\mathrm{T}}=[\mathbf{Q}_0^{\mathrm{T}}\ \mathbf{Q}_1^{\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{G}_{\mathrm{T}}=[\mathbf{G_0}\ \mathbf{G_1}]$ yields $\mathbf{Q}_{\mathrm{T}}=\left[-\mathbf{G}_0^{-1}\mathbf{G}_1\ \ \mathbf{I}_{2N_1+8N_2-7}\right]^{\mathrm{T}}\cdot \mathrm{Q}_1$, where $\mathbf{G_0}$ is an invertible matrix with dimension of 10×10. Letting $\boldsymbol{\Phi}=\left[-\mathbf{G}_0^{-1}\mathbf{G}_1\ \ \mathbf{I}_{2N_1+8N_2-7}\right]^{\mathrm{T}}$, the vibration equation (9) can be written as:

$$\left(\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\mathbf{\Phi}\right)\ddot{\mathbf{Q}}_1+\left(2\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{\Phi}}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{C}\mathbf{\Phi}\right)\dot{\mathbf{Q}}_1+\left(\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\ddot{\mathbf{\Phi}}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{M}\dot{\mathbf{\Phi}}+\mathbf{\Phi}^{\mathrm{T}}\mathbf{K}\mathbf{\Phi}\right)\mathbf{Q}_1=\mathbf{\Phi}^{\mathrm{T}}\mathbf{F}_{\mathrm{T}} \tag{10}$$

3 SOLUTIONS AND RESULTS

Parameters of a rope-guided system are presented in Table 1.

Table 1 Parameters of rope-guided hoisting System

Parameters	Descriptions	Values
m	Conveyance mass and loads	24000 [kg]
T_b	Average bottom tension of guiding rope	56.5 [kN]
ρ_0	Length density of hoisting ropes	6×2.62 [kg/m]
ρ_i	Length density of guiding rope	8.94 [kg/m]
$a_1(a_2)$, $b_1(b_2)$	Cross-sectional dimension of conveyance	0.58 [m], 1.095 [m]
L_i	Length of guiding rope	1012 [m]
H	Hoisting height	1000 [m]
v	Hoisting velocity	9.7 [m/s]

According to Article 388 of Coal Mine Safety Regulations [6], tension differences between guiding ropes are no less than 5% of their average tensions and tensions of two ropes near the mine shaft center are larger than that of the other two. Therefore, setting T_{b1} = 1.025T_b, T_{b2} = 1.075T_b, T_{b3} = 0.975T_b, T_{b4} = 0.925T_b as 5% tension differences, the system excitations from head sheave in w and u directions are set as $e_1(t) = A_1\sin\pi t$, $e_2(t) = A_2\sin\pi t$, respectively, where A_1 = 0.03 m and A_2 = 0.04 m are assumed. Lateral and torsional coupling vibrations are show as Fig. 2 under tension differences of guiding ropes. It can be seen from curves of $w_c(t)$ and $u_c(t)$ that there are only small differences in amplitude of lateral responses under equal tensions of guiding ropes in w or u directions, because there are only excitation amplitude difference of $w_c(t)$ and $u_c(t)$, and the torsional responses are too small to affect the lateral displacement.

Lateral and torsional coupling vibrations under different linear density of guiding ropes and different eccentricity of hoisting rope are shown in Fig. 3 and Fig. 4. The torsional responses are also small within 0.1m eccentricity installation error, while linear density differences of guiding ropes have an obvious effect on torsional vibration of conveyance. Since obvious torsional vibration can affect lateral vibration, linear density of guiding ropes should be well designed to satisfy sufficient space between conveyances and shaft wall.

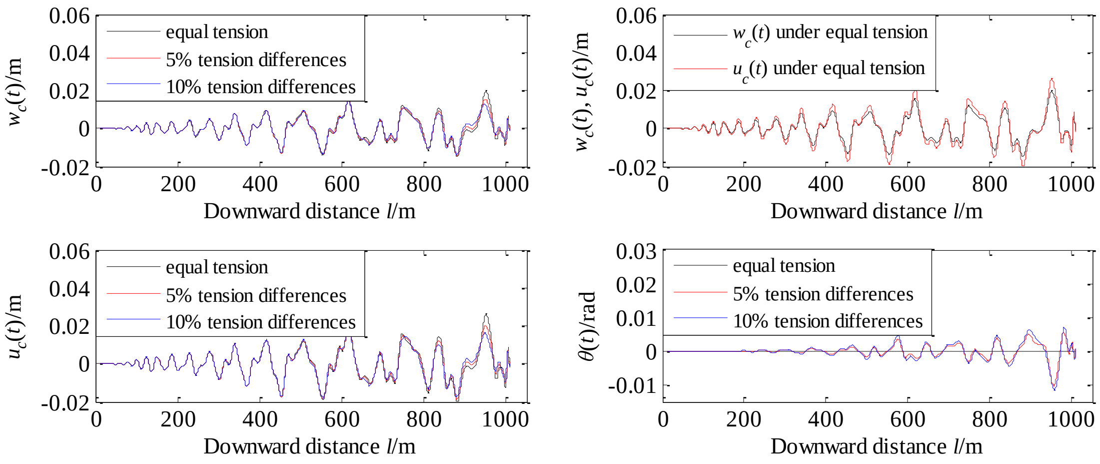

Figure 2 Vibration under tension differences of guiding ropes

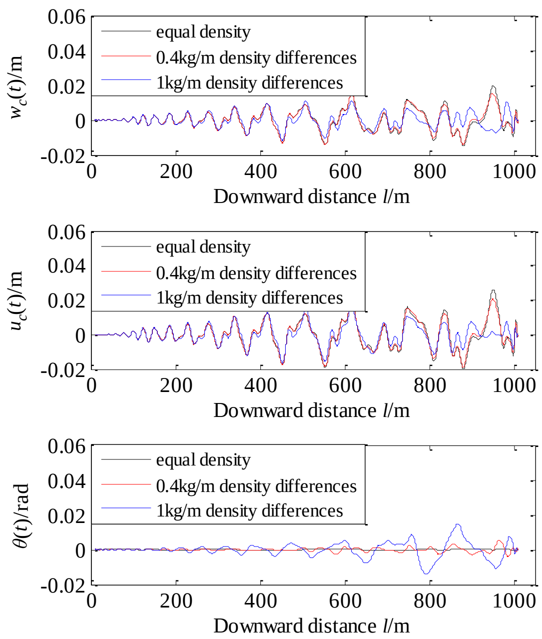

Figure 3 Vibration under liner density differences of guiding ropes

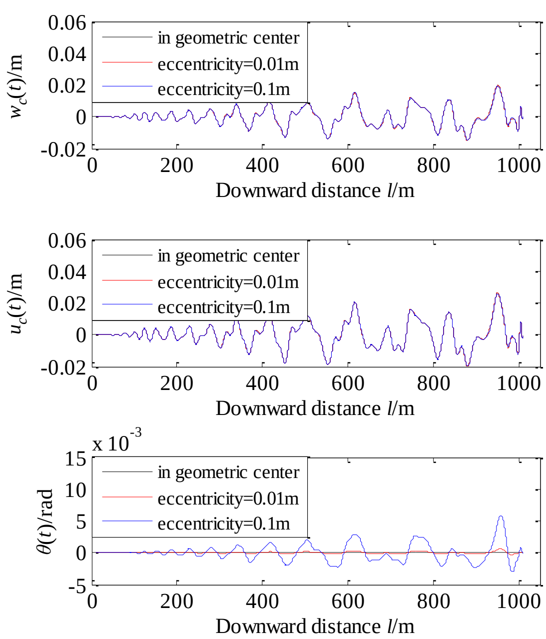

Figure 4 Vibration under eccentricity differences of hoisting ropes

4 CONCLUSIONS

Coupled vibration model of time-varying hoisting systems with four rope guides is built by energy method and the system vibration equation is established with Lagrange multiplier method under multiple constraint conditions. Lateral and torsional coupling responses are obtained under tension and density differences of guiding ropes as well as eccentricity between hoisting ropes and conveyance. These characteristics in some degree may provide guidelines for the design of hoisting system to prevent collision between conveyances and the shaft wall..

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