Modelling and Simulation of a Nonstationary High-Rise Elevator System to Predict the Dynamic Interactions Between Its Components

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Abstract. Lateral vibrations of the suspension and compensating ropes in a high-rise elevator system are induced by the building motions. When the elevator is in motion the length of the ropes change so that the natural frequencies vary, rendering the system nonstationary. In this scenario large displacements of the ropes occur when a passage through resonance takes place. Due to the nonlinear coupling, interactions between the in-plane and out of plane motions of the ropes occur. Furthermore, the car, counterweight and compensating sheave suffer from vertical vibrations due to the coupling with lateral vibrations of the ropes. This paper presents a mathematical model of a high-rise elevator system which can be used to predict the dynamic interactions taking place during its operation. The model is implemented in a high performance computational environment and the dynamic response of the system when the building is subjected to a low frequency sway, is determined through numerical simulation with the car following the kinematic profile dictated by the drive control algorithm. A case study is used to demonstrate resonance phenomena taking place during the operation of the system. The results predict a range of nonlinear dynamic interactions between the components of the elevator system, during travel and when the system is stationary.

1 INTRODUCTION

When one of the fundamental frequencies of the building structure coincides with the natural frequencies of the ropes in the elevator installation, large resonance whirling motions of the suspension and compensating ropes occur [1]. This results in impact loads taking place in the elevator shaft, leading to adverse dynamic behavior of the elevator system. When the elevator system is in motion transient/nonstationary resonance phenomena may take place. A nonstationary linear planar model of an elevator system was presented in [2] which was developed further in [3] to accommodate nonlinear modal interactions in a system consisting of a vertical rope of varying length moving at speed within a tall host structure subjected to a low frequency sway. The study presented in [4] involved the prediction of internal resonance behavior of an elevator system represented by a rope of time varying length translating vertically with a car modeled as a spring-mass system. In this paper a nonstationary model of a high-rise elevator system is developed. The system operates in a building host structure subjected to a low frequency sway. This model is then implemented in a high-performance computational platform to carry out numerical simulations in order to predict the dynamic interations between the building sway, the rope motions and the vibrations of the elevator components such as the car, compensating sheave, and counterweight. The effects of centrifugal forces and coriolis acceleration arising due to transportation motion are accounted for.

2 DESCRIPTION OF THE MODEL OF AN ELEVATOR SYSTEM

The model of an elevator system with a car of mass M_1, compensating sheave of mass M_2, and counterweight of mass M_3, is depicted in Fig. 1. The suspension and compensating ropes have mass per unit length m_1 and m_2, elastic modulus E_1 and E_2, and effective cross-section are A_1 and A_2, respectively. The parameter b_1 represents the distance measured from the bottom landing level to the center of the compensating sheave. The parameter b_2 denotes the distance measured from the center of the traction sheave to the center of the diverter pulley and h_0 represents the distance measured from the bottom landing level to the center of the traction sheave. The parameter h_{trav} is the height of travel of the elevator car. The parameter h_{car} is the height of the car. The parameter h_{cw} is the height of the counterweight. The parameter h_t is the position of the elevator car measured from the bottom landing level to the bottom of the elevator car vary with time according to the kinematic profile dictated by the drive control algorithm.

The lengths of the suspension rope and of the compensating rope are defined as follows. The length of the suspension rope at the car side measured from the center of the traction sheave to the the termination at the car crosshead beam is denoted by $L_1(t)$. The length of the compensating rope at the car side measured from the termination at the car bottom to the center of the compensating sheave is denoted as $L_2(t)$. The length of the compensating rope at the counterweight side measured from the termination at the counterweight to the center of the compensating sheave is denoted by $L_3(t)$. The length of the suspension rope at the counterweight side measured from the center of the diverter pulley to the termination at the counterweight end is denoted by $L_4(t)$. The mass moment of inertia of the diverter pulley and the short stretch of the suspension rope between the pulley and the traction sheave is neglected in the simulation model. They vary with time according to the kinematic profile dictated by the dive control algorithm.

The response of the elevator ropes subjected to dynamic loading due to the building sway are represented by the lateral in-plane and the lateral out of plane displacements denoted as $V_i(x_i(t),t)$ and $W_i(x_i(t),t)$ where the subscripts $i=1,2,3,4$ correspond to the rope sections of length L_1, L_2, L_3, and L_4, respectively. The lateral in-plane and lateral out of plane motions of the ropes are coupled with their longitudinal motions that are denoted as $U_i(x_i(t),t)$. The longitudinal motions of the car, compensating sheave and counterweight are denoted as $U_{CR}(t)$, $U_{CS}(t)$, and $U_{CW}(t)$, respectively.

3 VIBRATION MODEL

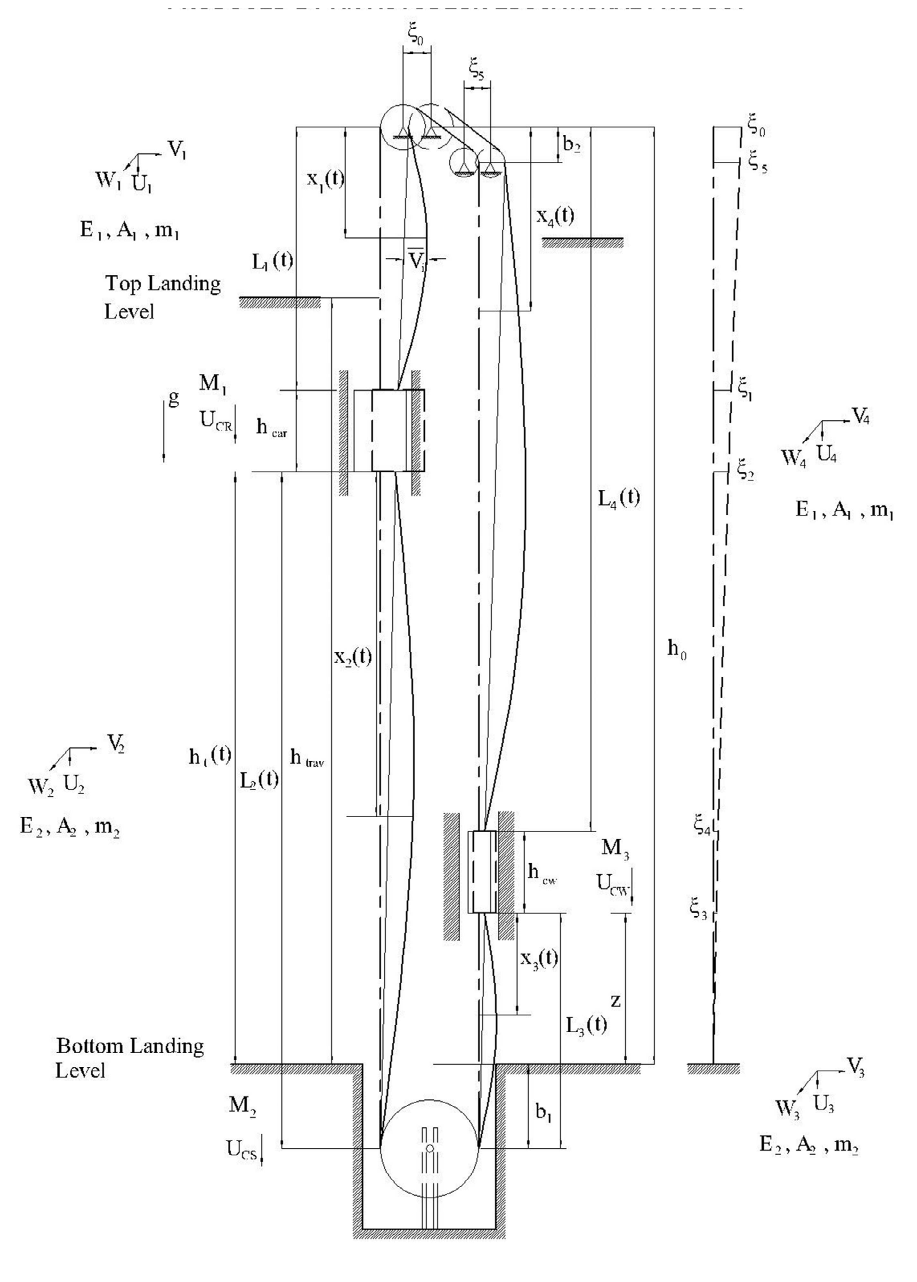

Figure 1 Elevator system

The axial Green's strain measure representing stretching of the rope section *i* is given as

$$\varepsilon_i = U_{ix} + \frac{1}{2}\left(V_{ix}{}^2 + W_{ix}{}^2\right). \tag{1}$$

where $(\ )_x \equiv \frac{\partial(\ )}{\partial x}$. The equations governing the undamped dynamic displacements $U_i(x_i(t),t)$, $V_i(x_i(t),t)$, $W_i(x_i(t),t)$, $U_{CR}(x_{CR},\ t)$, $U_{CS}(t)$, and $U_{CW}(x_{CW},\ t)$ can be developed by applying Hamilton's principle, which yields

$$m_i V_{itt} + 2m_i v V_{xt} + m_i v^2 V_{xx} + m a_c V_{ix} - T_{ix} V_{ix} - E_j A_j \varepsilon_{ix} V_{ix} - T_i V_{ixx} - E_j A_j \varepsilon_i V_{ixx} = 0\,. \tag{2}$$

$$m_i W_{itt} + 2 m_i v W_{xt} + m_i v^2 W_{xx} + m a_c W_{ix} - T_{ix} W_{ix} - E_j A_j \varepsilon_{ix} W_{ix} - T_i W_{ixx} - E_j A_j \varepsilon_i W_{ixx} = 0 . \tag{3}$$

$$m_i U_{itt} - E_j A_j \varepsilon_{ix} = 0 . \tag{4}$$

$$M_1 \ddot{U}_{CR} + M_1 a_c - M_1 g + T_1(L_1) + E_1 A_1 (\varepsilon_1)\big|_{x=L_1} - T_2(0) - E_2 A_2 (\varepsilon_2)\big|_{x=0} = 0 . \tag{5}$$

$$M_2 \ddot{U}_{CS} - M_2 g + T_2(L_2) + E_2 A_2 (\varepsilon_2)\big|_{x=L_2} + T_3(L_3) + E_2 A_2 (\varepsilon_3)\big|_{x=L_3} = 0 . \tag{6}$$

$$M_3 \ddot{U}_{CW} + M_3 a_{cw} - M_3 g + T_4(L_4) + E_4 A_4 (\varepsilon_4)\big|_{x=L_4} - T_3(0) - E_2 A_2 (\varepsilon_3)\big|_{x=0} = 0 . \tag{7}$$

where T_{ix} represent the mean tension of each stretch of rope, g is the acceleration of gravity, a_c is the acceleration of the car, a_{cw} is the acceleration of the counterweight, $x_i(t)$ represent the spatial coordinate corresponding to the sections of the ropes of length $L_1(t)$, $L_2(t)$, $L_3(t)$, and $L_4(t)$ in time t, respectively and $(\ )_t \equiv \frac{\partial(\ )}{\partial t}$ and an overdot denotes the derivative with respect to time, v represents the velocity defined according to the kinematic profile of the car.

From here on the procedure described in [1] is the same. The steps consist in neglecting the longitudinal inertia of all ropes can be neglected in Eq. (4) so that the model is reduced to two equations for each section of the suspension and compensating ropes. The Galerkin method is used to determine an approximate solution to the nonlinear partial differential equations of motion, the boundary conditions given by Hamilton Principle in [1] and the overall lateral in-plane and lateral out of plane displacements of each rope, with the following finite series:

$$\bar{V}_i(x_i, t) = \sum_{r=1}^{N} \phi_{ir}(x_i) q_{ir}(t) . \tag{8}$$

$$\bar{W}_i(x_i, t) = \sum_{r=1}^{N} \phi_{ir}(x_i) \, \mathrm{z}_{ir}(t) . \tag{9}$$

where $\phi_{ir}(x_i) = \sin\left(\frac{n\pi}{L_i} x_i\right)$; $r = 1, 2, 3, ..., N$; with N denoting the number of modes, are the natural vibration modes of the corresponding i^{th} rope and $q_{ir}(t)$ and $z_{ir}(t)$; $r = 1, 2, ..., N$ represent the lateral in-plane and lateral out of plane modal displacements, respectively. The final set of 4xN ordinary differential equations for the lateral in plane and lateral out of plane direction are the following

$$\ddot{\mathrm{q}}_{ir}(t) + 2\zeta_{ir}\omega_{ir}(t)\dot{\mathrm{q}}_{ir}(t) + \sum_{p=1}^{N} \bar{K}_{irp}(t) \mathrm{q}_{ip}(t) = \bar{f}_{ir}^{\,q} + N_{ir} \, \mathrm{q}_{ir}(t) . \tag{10}$$

$$\ddot{z}_{ir}(t) + 2\zeta_{ir}\omega_{ir}(t)\dot{z}_{ir}(t) + \sum_{p=1}^{N} \bar{K}_{irp}(t) z_{ip}(t) = \bar{f}_{ir}^{\,z} + N_{ir} z_{ir}(t) . \tag{11}$$

The modal damping represented by the ratios ζ_{ir} and the undamped time varying natural frequencies of the element ω_{ir}. The $\bar{K}_{irp}$ is the stiffness matrix, $\bar{f}_{ir}^{q}$ and $\bar{f}_{ir}^{z}$ represent the excitation force terms and N_{ir} are the nonlinear terms.

Similarly, the equations of motion for the car, compensating sheave, and counterweight from Eq. (5) to Eq. (7) are transformed into the modal coordinates using the transformation

$$\vec{U} = [Y]\vec{S} \tag{12}$$

where $\vec{U} = [U_{CR} \quad U_{CS} \quad U_{CW}]^T$ and $\vec{S} = [S_{CR} \quad S_{CS} \quad S_{CW}]^T$ is a vector of modal-coordinates corresponding to the system comprising the car, compensating sheave, and counterweight, respectively. If [*Y*] is the mass-normalized mode shape matrix, the following set of equations describing the vertical response of the car, compensating sheave and counterweight: in terms of the modal parameters

$$\ddot{S}_{CR}(t) + 2\zeta_{CR}\omega_{CR}(t)\dot{S}_{CR}(t) + \omega_{CR}^2(t)S_{CR}(t) = \left(\vec{Y}^{(1)}\right)^T\left(\vec{F} + \vec{\eta}\right). \tag{13}$$

$$\ddot{S}_{CS}(t) + 2\zeta_{CS}\omega_{CS}(t)\dot{S}_{CS}(t) + \omega_{CS}^2(t)S_{CS}(t) = \left(\vec{Y}^{(2)}\right)^T\left(\vec{F} + \vec{\eta}\right). \tag{14}$$

$$\ddot{S}_{CW}(t) + 2\zeta_{CW}\omega_{CW}(t)\dot{S}_{CW}(t) + \omega_{CW}^2(t)S_{CW}(t) = \left(\vec{Y}^{(3)}\right)^T\left(\vec{F} + \vec{\eta}\right). \tag{15}$$

where ζ_{CR}, ζ_{CS}, ζ_{CW} and ω_{CR}, ω_{CS}, ω_{CW} denote the modal damping ratios and the natural frequencies of the car, compensating sheave and counterweight, respectively, and $\vec{Y}^{(i)}$ is the *i*th mode shape vector. The $\vec{F} = \begin{bmatrix} \bar{F}_{CR} \\ \bar{F}_{CS} \\ \bar{F}_{CW} \end{bmatrix}$ is the excitation vector, and the $\vec{\eta} = \begin{bmatrix} \eta_{CR} \\ \eta_{CS} \\ \eta_{CW} \end{bmatrix}$ is a vector with components representing the nonlinear couplings with the lateral motions of the ropes.

4 CASE STUDY

A case study will be presented to illustrate the dynamic performance of an elevator system. The system comprises seven ($n_1 = 7$) steel wire suspension ropes and four ($n_2 = 4$) steel wire compensating ropes of mass per unit length $m_1 = 0.723$ kg/m and $m_2 = 1.1$ kg/m, having modulus of elasticity $E = 54535$ N/mm^2 and nominal diameters $d_1 = 13$ mm and $d_2 = 16$ mm, respectively. The modal damping ratios for the ropes are assumed as 0.3% across all modes and 10% across all the lumped modes. The height measured from the ground floor level to the center of the traction sheave is $h_0 = 88.875$ m, the car and counterweight height is $h_{cw} = h_{car} = 4.00$ m, travel height $h_{trav} = 80.70$ m, the car mass with full load is $M_1 = 4400$ kg, the mass of the compensating sheave is $M_2 = 600$ kg, and the mass of the counterweight is $M_3 = 3600$ kg. The high rise building is excited harmonically in the lateral in-plane at a frequency equal to the natural frequency of the compensating rope when the car is passing through the middle of the travel height. In the lateral out of plane the building is excited at a much lower frequency. The elevator car is positioned at the bottom landing level and starts ascending to the top landing level with an acceleration of a_c=-1.1m/s^2 and the counterweight goes downward with an acceleration of a_{cw}=1.1m/s^2. Both the car and counterweight achieve a maximum speed of v=8m/s. The height measured from the bottom landing level to the center of the compensating sheave is given as $b_1 = 2.02\,\text{m}$ and the height from center of the traction sheave to the center of the diverter pulley is $b_2 = 0.80\,\text{m}$. The results will be illustrated using computer animation and to demonstrate the nonlinear dynamic interactions between the components of the elevator system, during travel and when the system is stationary.

5 CONCLUSION

The equations of motion of a nonstationary elevator system following the kinematic profile dictated by the drive control algorithm comprising an elevator car, compensating sheave, counterweight, with suspension and compensating ropes excited by the high rise building motions are derived in this paper. These equations accommodate the nonlinear effects of the rope stretching in the lateral in-plane and the lateral out of plane directions. This model is used to predict the response of the system. While the motions of the structure are small, the rope is experiencing large lateral whirling motions. If the response of the ropes continue to grow impact phenomena in the hoistway might occur which may lead to excessive vibrations of the car and damage to the system components.

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6 REFERENCES

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BIOGRAPHICAL DETAILS

Rafael Sanchez Crespo is a full-time PhD student at the University of Northampton, UK, with a scholarship co-funded by ThyssenKrupp Elevator AG and the University of Northampton. His PhD project concerns the effects of the dynamic responses of tall buildings on high-rise elevator systems. He has a Masters Degree in Structural Engineering from the Silesian University of Technology in Gliwice, Poland and a Bachelor Degree in Civil Engineering from the National Autonomous University of Honduras. He has earned several scholarships and academic awards during his studies.

Stefan Kaczmarczyk is Professor of Applied Mechanics at the University of Northampton. His expertise is in the area of applied dynamics and vibration with particular applications to vertical transportation and material handling systems. He has been involved in collaborative research with a number of national and international partners and has an extensive track record in consulting and research in vertical transportation and lift engineering. He has published over 90 journal and international conference papers in this field.

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