

# 3<sup>rd</sup> Symposium on Lift and Escalator Technologies

## Modelling and Simulation of the Dynamic Behaviour of Elevator Systems

Seyed Mirhadizadeh<sup>1</sup>, Stefan Kaczmarczyk<sup>2</sup>

The University of Northampton, Northampton, NN2 6JD, UK

<sup>1</sup>seyed.mirhadizadeh@northampton.ac.uk

<sup>2</sup>stefan.kaczmarczyk@northampton.ac.uk

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**Abstract.** An elevator represents a multi-body system deployed in buildings to provide vertical transportation. Vibration phenomena taking place in elevator and hoist installations may influence the dynamic performance of their components which in turn may affect ride quality of a lift car. Lateral and longitudinal vibrations of suspension ropes and compensating cables may result in an adverse dynamic behaviour of the entire installation. There is a need to predict the dynamic behaviour of elevator systems under various operating conditions. In particular, it is necessary to predict any possible failures that would require their shutdowns. This paper presents the results of work to develop adequate mechanical models of elevator systems in a multibody simulation software environment. Using these models an analysis can be performed to investigate the influence of design parameters on their performance. Simulation tests have then been carried out and the results are graphically presented through diagrams and animations, for a range of elevator parameters. Conclusions concerning their influence on elevator performance can then be formulated.

## INTRODUCTION

Dynamic simulation of multibody systems plays an important role in a wide range of fields, as in engineering applications, the main goal is to design and manufacture marketable products of high quality. Simulation analysis allows an engineer to simulate the dynamic behaviour of a product. Based on the results, the product design can be optimized prior to actual production. A product may contain mechanical, electrical, or other components. If mechanical components are allowed to move relative to one another, the product is called a multibody (MBD) system [1].

A MBD system is one that consists of solid bodies, or links, that are connected to each other by joints that restrict their relative motion. The study of MBD is the analysis of how mechanisms and systems move under the influence of forces, also known as forward dynamics. A study of the inverse problem, i.e. what forces are necessary to make the mechanical system move in a specific manner is known as inverse dynamics. Motion analysis is important because product design frequently requires an understanding of how multiple moving parts interact with each other and their environment [1,2]. An elevator represents a MBD system deployed in buildings to provide vertical transportation. Vibrations of elevator components may influence the dynamic performance of their components which in turn may affect ride quality of a lift car [3]. This paper presents the theory and MBD simulation results using mechanical models of an elevator system in a multibody simulation software environment. Using these models an analysis can be performed to investigate the influence of design parameters on their performance.

## CASE STUDY

### Theoretical model

A lift car of mass  $P = 1000$  kg is supported by a platform mounted within a sling on elastomeric isolation pads of combined stiffness coefficient  $k_p = 1160$  kN/m as depicted in Fig. 1. The sling mass is  $M = 400$  kg and the car – sling assembly is suspended on 4 steel wire ropes in 1:1 configuration. The ropes are of modulus of elasticity  $E = 0.85 \times 10^5$  N/mm<sup>2</sup>, mass per unit length  $m_r = 0.66$  kg/m, metallic (effective) area  $A_{eff} = 69$  mm<sup>2</sup> (see Table 1). The main propose of this case study is to determine the natural frequencies and modal vectors of the car-sling-rope assembly when the lift is stationary and the length of the ropes at the car side is  $L = 30$  m,.

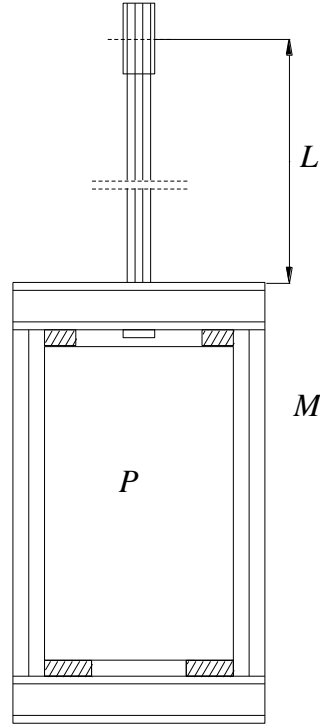


Figure 1. Elevator car-sling assembly and suspension system.

A simplified model of the system is illustrated in Fig. 2. It is evident that the model is essentially equivalent to a 2DOF system. The masses  $M_1$  and  $M_2$ , representing the sling and the car respectively, are constrained by two springs of constants  $k_p$  and  $k_e$  and they can move vertically so that their position is defined by the coordinates  $x_1$  and  $x_2$ , respectively. The equations of free undamped motion of the system given by Eq. 1 - 3 can be derived by the application of Newton's 2nd law [2].

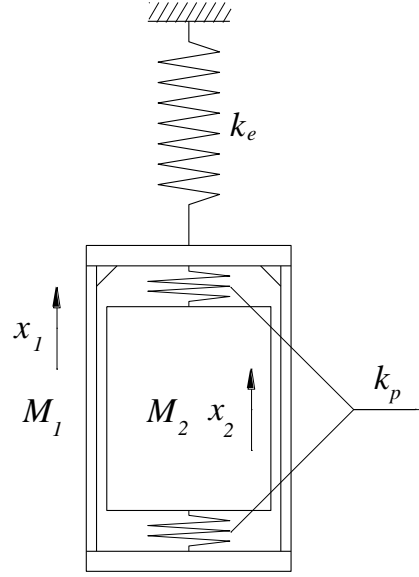


Figure 2. 2DOF model of a lift car – sling – suspension system.

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0} \quad (1)$$

where

$$\mathbf{M} = \begin{bmatrix} M_e & 0 \\ 0 & M_2 \end{bmatrix} \quad (2)$$

$$\mathbf{K} = \begin{bmatrix} k_e + k_p & -k_p \\ -k_p & k_p \end{bmatrix} \quad (3)$$

represent  $2 \times 2$  symmetric mass and stiffness matrices, where  $n_r$  is the number of suspension ropes. In this formulation the generalized coordinates are assembled in the displacement vector  $\mathbf{x} = [x_1(t), x_2(t)]^T$  and the right hand-side of Eq. 1 is a  $2 \times 1$  zero vector  $\mathbf{0} = [0, 0]^T$ . Assuming  $n_r = 4$  the effective stiffness of the suspension system is

$$k_e = n_r \frac{EA_{eff}}{L} \quad (4)$$

and the equivalent mass of the sling – suspension rope assembly is expressed as:

$$M_e = M_1 + \frac{n_r m_r L}{3} \quad (5)$$

Free undamped vibration of a single degree of freedom (SDOF) system is represented by a harmonic motion. Using the same approach in this model it can argued that the masses  $M_e$  and  $M_2$  move according to

$$\mathbf{x} = \mathbf{X} \cos(\omega t + \varphi); \mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (6)$$

where  $\omega$  is the natural frequency and  $\mathbf{X}$  represents a vector of modal amplitudes or shapes (the eigenvector). Thus, by assuming that both masses vibrate at the same frequency and are in phase but

have different amplitudes. Such a motion is referred to as synchronous and it is evident that the ratio between the two displacements remains constant throughout the motion so that

$$\frac{X_1}{X_2} = \text{const} \quad (7)$$

Inserting Eq. 6 into equation of motion Eq. 1 the following results

$$(\mathbf{K} - \omega^2 \mathbf{M}) \mathbf{X} = \mathbf{0} \Rightarrow \begin{bmatrix} k_e + k_p - \omega^2 M_e & -k_p \\ -k_p & k_p - \omega^2 M_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (8)$$

which represents two simultaneous homogenous algebraic equations in the unknowns  $X_1$  and  $X_2$  with  $\omega^2 \equiv \lambda$  playing the role of a parameter (referred to as an eigenvalue). The problem of finding the values of the parameter  $\lambda$  for which the above equation has a nonzero (nontrivial) solution is referred to as the eigenvalue problem. It is known from linear algebra that the above equation possess a nontrivial solution if the determinant of the coefficient matrix is zero

$$\Delta(\lambda) = \det \begin{bmatrix} k_e + k_p - \omega^2 M_e & -k_p \\ -k_p & k_p - \omega^2 M_2 \end{bmatrix} = 0 \quad (9)$$

Expanding the determinant in equation (11) yields the following characteristic equation (often referred to as frequency equation) for the unknown quantity  $\lambda \equiv \omega^2$

$$\lambda^2 - \left( \frac{k_e + k_p}{M_e} + \frac{k_p}{M_2} \right) \lambda + \frac{k_e k_p}{M_e M_2} = 0 \quad (10)$$

This expression represents a quadratic equation in  $\lambda$  and yields two positive, real roots (eigenvalues) as follows

$$\lambda_{1,2} = \frac{1}{2} \left( \frac{k_e + k_p}{M_e} + \frac{k_p}{M_2} \right) \mp \sqrt{\left( \frac{k_e + k_p}{M_e} + \frac{k_p}{M_2} \right)^2 - 4 \frac{k_e k_p}{M_e M_2}} \quad (11)$$

The corresponding natural frequencies are then found to be  $\omega_{1,2} = \sqrt{\lambda_{1,2}}$ . Thus, there are two vectors of amplitudes (mode shapes or eigenvectors) corresponding to each natural frequency:

$$\begin{aligned} \omega_1 &\Rightarrow \mathbf{X}^{(1)} \\ \omega_2 &\Rightarrow \mathbf{X}^{(2)} \end{aligned} \quad (12)$$

to be determined from the following equations

$$(\mathbf{K} - \omega_1^2 \mathbf{M}) \mathbf{X}^{(1)} = \mathbf{0} \Rightarrow \begin{bmatrix} k_e + k_p - \omega_1^2 M_e & -k_p \\ -k_p & k_p - \omega_1^2 M_2 \end{bmatrix} \begin{bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (13)$$

$$(\mathbf{K} - \omega_2^2 \mathbf{M}) \mathbf{X}^{(2)} = \mathbf{0} \Rightarrow \begin{bmatrix} k_e + k_p - \omega_2^2 M_e & -k_p \\ -k_p & k_p - \omega_2^2 M_2 \end{bmatrix} \begin{bmatrix} X_1^{(2)} \\ X_2^{(2)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (14)$$

The mode shapes / eigenvectors can be then normalized to satisfy the following condition

$$(\alpha_i \mathbf{X}^{(i)})^T \mathbf{M}(\alpha_i \mathbf{X}^{(i)}) = 1 \Rightarrow \mathbf{Y}^{(i)T} \mathbf{M} \mathbf{Y}^{(i)} = 1 \quad (15)$$

Parameter	Value	Unit
$M_I$	400	kg
$M_2$	1000	kg
$m_r$	0.66	kg/m
$E$	85000	N/mm <sup>2</sup>
$A_{eff}$	69	mm <sup>2</sup>
$n_r$	4	m
$k_p$	1160	kN/m
$L$	30	m

Table 1 Fundamental parameters of the system

$$\omega_1 = 20.0083 \frac{\text{rad}}{\text{s}} (3.1844 \text{ Hz}) \Rightarrow \mathbf{Y}^{(1)} = [-0.0190, -0.0291] \quad (16)$$

$$\omega_2 = 72.8977 \frac{\text{rad}}{\text{s}} (10.4064 \text{ Hz}) \Rightarrow \mathbf{Y}^{(2)} = [-0.0445, 0.0124] \quad (17)$$

The mode shapes are plotted in Figure 3. They illustrate that when the system vibrates in its first mode the amplitude of the second mass is greater than that of the first mass. The motions of the two masses are in phase. When the system vibrates in its second mode the amplitude of the first mass is greater and the magnitudes have opposite signs. Thus, the motions are 180° out of phase. It can be noted that one point/ section of the second spring remains stationary at all times; such a point is referred to as a node.

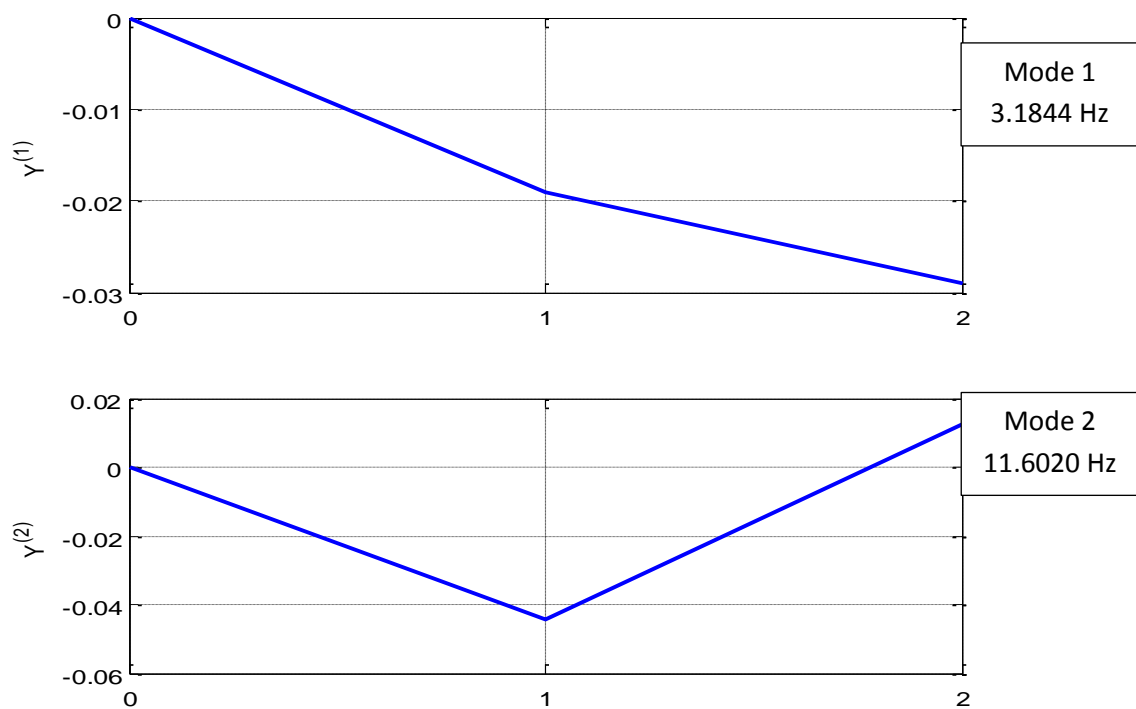


Figure 3. Mode shapes.

It is evident, that in the first mode the sling and the car move in phase. The second (higher) mode is mainly associated with the motion of the sling – suspension rope assembly (with the amplitude of the car close to zero).

### MBD simulation model and results

Using ADAMS/Vibration tools, vibrations of the system represented by the model can be studied. With MBD simulations in ADAMS, physical tests on shakers can be replaced with virtual prototype testing. Noise and vibration are critical factors in the performance of many mechanical designs, with MBD simulation the forced response of a model in the frequency domain over different operating points, evaluate frequency response functions for magnitude and phase characteristics, tabulate contribution of model elements to kinetic, static, and dissipative energy distribution in system modes or animate forced response and individual mode response can be investigated.

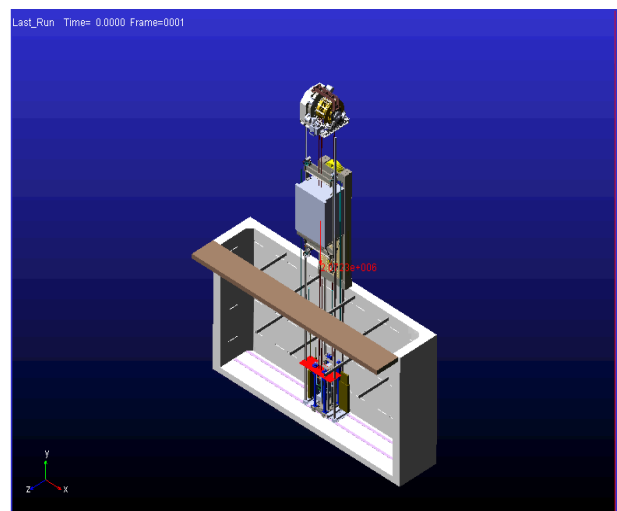
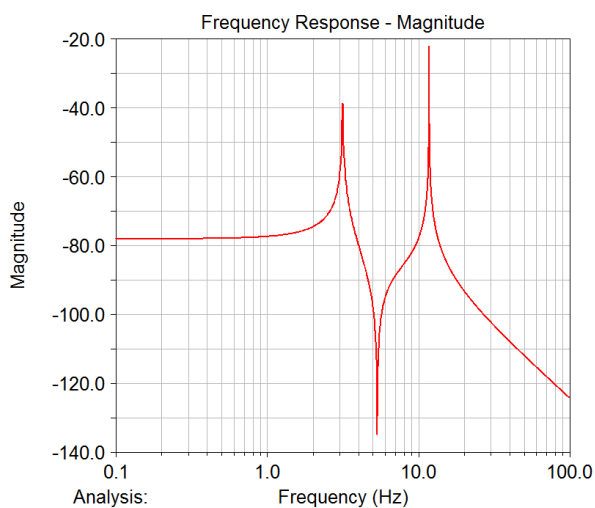


Figure 4. Car-sling-suspension rope simulation model and modes.

Fig. 4 illustrates the usefulness of the solution of the eigenvalue problem. This simulation can demonstrate how the system behaves and helps engineers to associate the natural (resonance) frequencies with the individual components of the system. Fig. 5 and Fig. 6 illustrate the modal behaviour of the car-sling-suspension assembly. The simulation test returns the same numerical values of the natural frequencies and the natural modes as calculated in equations (16-17) and the modal behaviour is illustrated through computer animation. This information is valuable to improve performance and control of any specific mode in order to suppress excessive vibrations in the system.

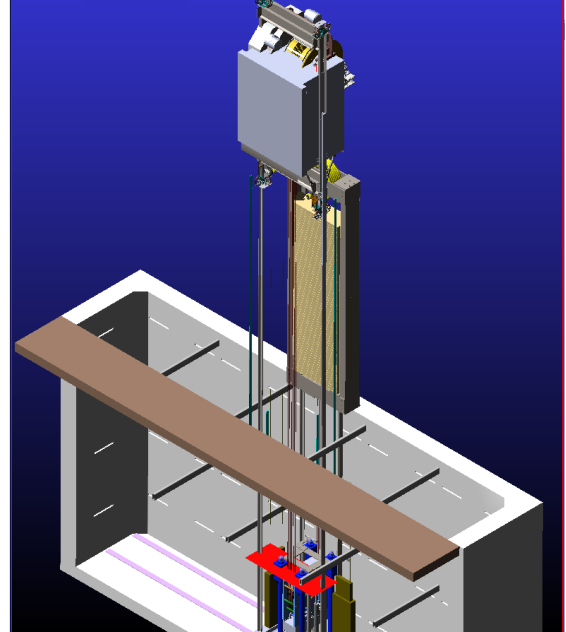
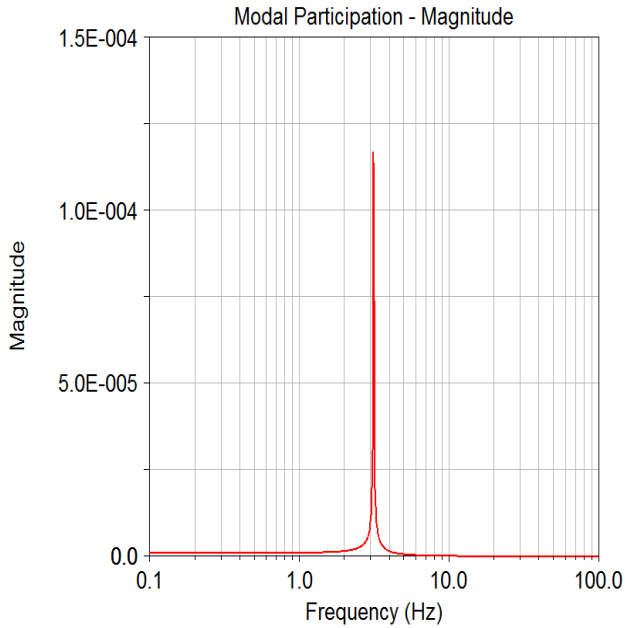


Figure 5. The 1<sup>st</sup> natural frequency and mode simulation and behaviour of the car-sling-suspension MBD system at 3.1844 Hz .

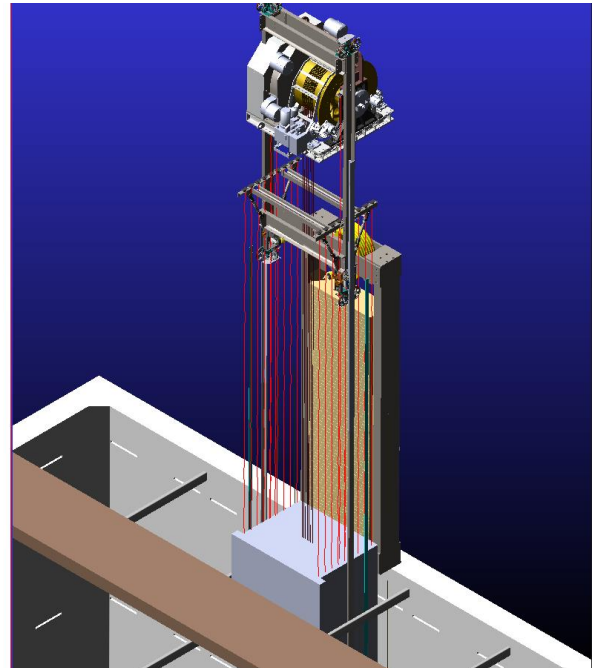
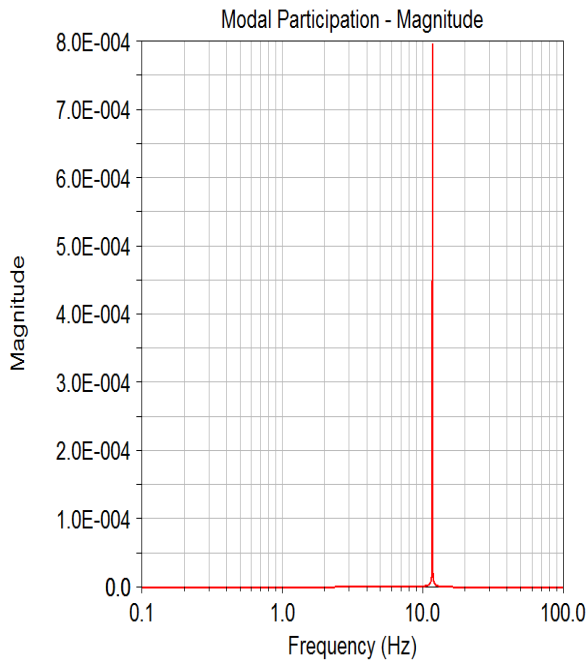


Figure 6. The 2<sup>nd</sup> natural frequency and mode simulation and behaviour of the car-sling-suspension MBD system at 11.6020 Hz

## **CONCLUSION**

An elevator represents a complex MBD system with its dynamic characteristics varying during the travel. MBD modelling and computer simulation techniques can be employed to investigate the dynamic behaviour of the elevator system and its components. However, the models and techniques should be checked through the application of benchmark problem tests and experimental validation so that the models can be used to make predictions with sufficient confidence.

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## **REFERENCES**

- [1] P. E. Nikravesh, *Planar Multibody Dynamics: Formulation, Programming, and Applications* (2008)
- [2] S. Kaczmarczyk, LIFM010DL Dynamics and Vibrations, MSc Lift Engineering Learning Materials, The University of Northampton (2006).
- [3] G.R. Strakosch, *The Vertical Transportation Handbook*. John Wiley, New York (1998).